Delay Minimization for Data Transmission in Wireless Power Transfer Systems

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***Abstract*—Radiative wireless power transfer (WPT) is a promising technique to power wireless devices' transmission. In a resource-limited device, receiving energy and transmitting data cannot operate at the same time because they share the same spectrum or hardware. This paper studies the problem for a wireless device to decide when to harvest energy, when to deliver data, and what transmission rate to use. Distinct from the most existing works, we focus on delay minimization in transmitting a sequence of data packets over a point-to-point channel, which is critical for time-sensitive applications. Since the battery is capacitated, the device must repeatedly switch between harvesting energy and transmitting data. For the offline case where packet information is known before scheduling, a surprising result is discovered that for all (energy receiving and data transmitting) cycles, except the last one, the optimal transmission rate should be a constant which is called the *wOPT rate*. Based on this discovery, the offline delay minimization problem is optimally solved. For the online case where packets arrive dynamically without prior information, we propose a simple online algorithm: using the *wOPT rate* to transmit whenever both energy and data are ready. It is proved to be 1.16-competitive if the battery is initially empty, namely, its delay is less than 1.16 times the offline optimal delay for any given packet set. When the battery is with arbitrary initial energy, simulation results show that the performance is near optimal. The discovery of the *wOPT rate* reveals an essential property of WPT and is expected to be significant in solving other related problems.**

***Index Terms*—Wireless power transfer, delay minimization, data transmission, online algorithm, competitive ratio, optimal transmission rate.**

I. Introduction

WITH the popularity of the Internet of Things and Smart Cities, wireless devices are widely deployed which are typically powered by battery. However, current wireless device batteries are further from satisfactory, which are typically large in size, heavy in weight, small in capacity, and slow to charge. Wireless power transfer (WPT) provides an alternative option to overcome these disadvantages. WPT enabled devices are more user-friendly, more cost-effective, more environmental preserving and sometimes essential [1].

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In industry, groups and companies have already been working on the commercialisation and standardization of WPT techniques. For instance, *Qi* [2] is a new WPT interface standard, which is becoming increasingly popular; *WiTricity* is a now one of the most important company dedicated to WPT who holds an important patent for it [3]. A large number of multinational corporations such as Apple, Samsung, Intel and Toyota have been involving in the *Qi* standard or *WiTricity* licensing. Consumer products are being available quite recently on the market [4], [5].

In academia, increasing interests are turning to WPT. Most recently, a team lead by Assawaworrarit *et al.* [6] proposed a novel WPT method that does not require any tuning, enable high efficiency charging to moving devices. One of the most significant work in networking field is by Liu *et al.* [7]. They design and make a new type of battery-free device that communicates with each other by energy harvested from television broadcast signals. A most recent work by Talla *et al.* [8] builds the first *power over Wi-Fi* system that delivers power via commercially available Wi-Fi chipsets. Such system can provide far field wireless power without compromising the network's communication performance.

In a resource limited device, receiving energy and transmitting data do not operate at the same time for many reasons, *e.g.,* one antenna is shared by the two modules [9], [10], limited bandwidth is shared by the two operations [11]. Ju and Zhang [12] study a new type of access point called hybrid data-and-energy access point (H-AP), which provides wireless energy to user devices and collects information from them. Since the power transfer is in the downlink (DL) while the data transmission is in the uplink (UL), they propose a 'harvest-then-transmit' protocol to coordinate the two operations to maximize network throughput. Resource allocation problems based on 'harvest-then-transmit' protocol are further studied in a large-scale wireless powered communication network by Che *et al.* [13] and for H-APs equipped with large number of antennas by Yang *et al.* [14]. All these works study the WPT problem from the point view of network throughput maximization. However, guaranteeing the maximum network throughput does not necessarily guarantee a specific user device's packet transmission delay, especially when these packets arrive dynamically. In many real world scenarios, the data transmission delay is required as a part of quality of service (QoS) for time-sensitive applications.

In this paper, we investigate a fundamental scheduling problem for a wireless device such that a sequence of dynamically arrived data packets can be transmitted with the minimum delay. As previous works [12]–[14], we adopt the 'harvest-then-transmit' protocol. The transmitter has to decide 1) when to receive energy and when to deliver data, 2) what transmission rate should be used to deliver data in each transmission period, 3) how often to repeat the (energy receiving, data transmitting) cycles? The ultimate goal is to minimize the completion time with all packets transmitted. We assume the most fundamental point-to-point single-user additive White Gaussian Noise (AWGN) channel for data transmission.

In our research, we need to address several challenges. According to Shannon-Hartley Theorem on wireless channel capacity, a low transmission rate is preferred to save energy, while a high rate is preferred to shorten transmission delay. Therefore, a major challenge lies on the trade-off between the following two strategies. On one hand, the more time assigned to receive power, the more energy is charged to the device allowing a higher transmission rate and shorter deliver time. On the other hand, the more time allocated to deliver data, the less energy is required and less time is needed to charge the device. To minimize the total time on receiving power and sending data, the best trade-off must be found.

Another challenge is that the device battery has a limited capacity. When it is full, no more energy can be added; while it is empty, no data can be transmitted. Therefore, the transmitter must alternatively change its operations, from charging the battery to sending data, and vice versa. This adds one more difficult factor to our problem.

The contributions are summarized as follows.

- We formally define the delay minimization scheduling problem for the wireless device with WPT capability.
- We discover that although the optimal switch time for each (energy receiving, data transmitting) cycle depends on the initial energy and packet sizes, the optimal transmission rate for all cycles except the last one is constant. Such rate is called the *wOPT rate*.
- Based on the *wOPT rate*, we design an optimal scheduling algorithm to solve the offline delay minimization problem. In which, we determine the optimal switching points until the very last cycle which needs higher rates to speed up the completion.
- For the online case where packets arrive dynamically without prior information, we propose a simple online algorithm: using the *wOPT rate* to transmit whenever energy and data are ready. It is proved to be 1.16-competitive if the battery is initially empty, namely its delay is less than 1.16 times the offline optimal delay for any packet set.
- The proposed online algorithm is evaluated for both empty initial battery case and arbitrary initial energy case, and simulation results show the performance is near optimal.
- The discovery of the *wOPT rate* reveals an essential property of WPT, thus is expected to be significant in solving other related scheduling problems in the field of WPT.

The organization of this paper is as follows. Related works are introduced in Section II. Section III formally defines the delay minimization scheduling problem. The *wOPT rate* is introduced in Section IV. Section V studies the offline problem, and utilizes the *wOPT rate* to optimally solve this problem. An online algorithm is proposed and analyzed in Section VI followed by simulations in Section VII that show its efficiency. Different from our preliminary work [15], the correctness proof for the offline algorithm is completely rephrased with improved logic and a competitive ratio is derived for the online algorithm. Besides, much more simulate results are now provided. Section VIII concludes this paper.

II. Related Work

Most related works focus on maximizing throughput in designing data transmission scheduling algorithms for wireless powered wireless devices. Ju and Zhang [12] are among the first group of researchers to investigate the throughput maximization problem for a given period and propose the 'harvest-then-transmit' protocol. They observed a tradeoff on time allocation for charging and sending: since the total time is given and fixed, on one hand, a longer charging time leads to more energy charged and a higher transmission rate in a shorter sending duration, on the other hand, a shorter charging duration results in less energy charged and a lower transmission rate in a longer sending duration. They have presented an optimal time allocation method to maximize the throughput. Liu *et al.* [28] extend this work by considering the fairness for multiple users in maximizing throughput and Zewde and Gursoy [27] extend this result by allowing QoS constraints. Zhao *et al.* [29] proposed a numerically searching technique to solve the same throughput maximization problem. All these works focus on finding the optimal time allocation to maximize throughput, while our work concentrates on computing the optimal transmission rate to minimize transmission delay. Recently, Chi *et al.* [30] study the problem of minimization of transmission completion time in wireless powered communication networks. The major difference is we deliver mulitple packets over a point-to-point channel, while they consider a one-to-many channel, and each of the clients has one data block to transmit.

Other research works study the throughput maximization problem in wireless powered communications assuming WPT in downlink and wireless information transfer in uplink can operate simultaneously. Morsi *et al.* [32] investigate transmission policies for point-to-point wireless powered communication. They propose two online transmission policies that require no knowledge of the uplink CSI and the energy harvesting profile of the downlink. In their solutions, they study the infinite and finite battery capacity cases respectively. Lee *et al.* [33] investigate the optimal energy and time allocation problem aiming at maximizing the uplink sum rate for multiple users. They also distinguish two cases for the battery capacity, *e.g.,* the infinite and finite capacity energy storage cases, and studied respectively. More related works on WPT can be found in a recent survey given by Zeng *et al.* [1].

Minimizing the transmission delay is another common goal in wireless transmission rate scheduling. In traditional battery powered wireless communication, it has been extensively studied [17]–[21]. Prabhakar *et al.* [17] and Uysal-Biyikoglu *et al.* [18] are among the first group to study the energy minimization problem for delivering a set of packets before a common deadline. They propose a *lazy* schedule to optimally solve the offline problem. Zafer and Modiano [19], [20] further generalize the problem to allow individual packet deadlines provided they follow the same order packets arrive. Most recently, Shan *et al.* [21] solve the energy minimization problem that allows arbitrary individual packet deadlines. They present the novel Densest Interval First (DIF) policy to address the offline optimal problem.

The delay minimizing transmission rate scheduling problem has also been investigated in energy harvesting systems [16], [22]–[24], [31], [34]. Yang and Ulukus [16], [22] consider the delay minimization problem for harvesting enabled channels assuming all harvesting events are pre-determined and take no time to receive the energy. They have obtained the offline optimal scheduling algorithm. Tutuncuoglu and Yener [23], [24] extend their work to let the battery have a limited capacity. Shan *et al.* [34] study the energy consumption minimization problem for an energy harvesting transmitter, they allow packets to have individual deadlines. The Truncation method is proposed to handle the individual deadline case optimally. Arafa *et al.* [31] investigate the problem to minimize the average delay experienced by the bits. The essential difference between energy harvesting technique [16], [22]–[24], [34] and the RF WPT technique lies in how energy is charged into batteries. Yang and Ulukus [16] assume energy is harvested from nature (solar, vibration, thermoelectric) by separate hardware, so they are charged into battery instantly consuming no time. While we assume energy is harvested from radio frequency (RF) by the same hardware used for transmitting in a resource limited device. As a result, receiving energy and transmitting data both take times, and do not operate at the same time as [9]–[14].

III. Problem Formulation

A. System Model

Suppose a WPT system consists of a receiver and a wireless powered transmitter. Let $P = \{P_1, P_2, \ldots, P_n\}$ be a set of n packets to be transmitted from the transmitter to the receiver, as shown in Fig. 1. Each packet P_i has a size B_i, an arrival time a_i, and is denoted as $P_i(B_i, a_i)$. We assume each packet has a distinct arrival time such that $a_1 < a_2 < \ldots < a_n$. If two or more packets arrive at the same time, we combine them into a single packet with its size being the sum of all sizes in these packets. The transmission of packet P_i can start only after its arrival time a_i. This is called the *causality constraint* [16].

The wireless transmitter is capable of receiving energy wirelessly via WPT technique. When receiving energy, its

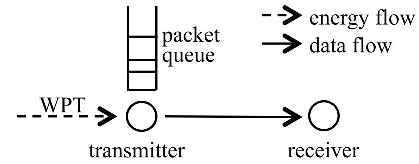


Fig. 1. A wireless powered transmission system.

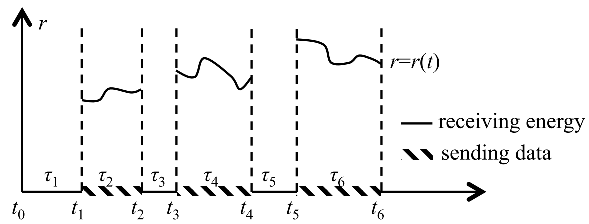


Fig. 2. Charging phases, sending phases and cycles.

battery is charged; the received energy then is used to send data at a later time. We therefore define the *charging phase* and *sending phase*, respectively. These two phases form a (energy receiving, data transmitting) cycle. Following previous work [12]–[14], a transmitter can either be in the *charging phase* or be in the *sending phase*, but not in both. The transmitter switches between the two phases alternatively according to a scheduling algorithm until all data packets are completely delivered.

Suppose there are m cycles. Thus, there are $2m$ phases and $2m$ switches, which occur at time instances $\{t_1, t_2, \ldots, t_{2m}\}$, $0 < t_1 < \ldots < t_{2m}$. The $2m$ phases are labeled from 1 to $2m$. Phase i starts from time t_{i-1} and ends at t_i. Its length is denoted as τ_i, *e.g.,* $\tau_i = t_i - t_{i-1}$. Note that we assume $t_0 = 0$. When no ambiguity arises, we also use the notation τ_i to denote Phase i. Phase $2i-1$ (τ_{2i-1}) is a charging phase, and Phase $2i$ (τ_{2i}) is a sending phase, $i = 1, 2, \ldots, m$. Note, we assume the last phase is a *sending phase*, this is because if it is otherwise a *charging phase*, we can delete it without affecting the transmission completion time. Therefore, the set $\{t_i\}$ is called the *switch points*, as shown in Fig. 2.

Let p be the energy transfer speed (amount of energy received per time unit) during the charging phases. The speed is assumed to be a constant speed.

B. Problem Formulation

Let $H(t)$ be the total energy charged into the battery in duration $[0, t]$. We can calculate $H(t)$ as follows.

$$H(t) = \begin{cases} \sum_{j=0}^{i-1} \tau_{2j+1} p & \text{for } t_{2i-1} \le t < t_{2i} \\ \sum_{j=0}^{i-1} \tau_{2j+1} p + (t - t_{2i}) p & \text{for } t_{2i} \le t < t_{2i+1} \end{cases}$$

During the *sending phases*, it is assumed that the transmitter can adaptively change its transmission rate.

Definition 1: The transmission rate function $r(t) : \mathbb{R}_{\ge 0} \to \mathbb{R}_{\ge 0}$ is defined as the transmission rate at time t.

We hence denote the transmission rate as a function of time

$$r(t) \begin{cases} = 0 & t \text{ in } \tau_1, \tau_3, \ldots, \tau_{2m-1} \\ \neq 0 & t \text{ in } \tau_2, \tau_4, \ldots, \tau_{2m} \end{cases} \quad (1)$$

The transmission rate $r(t)$ is related to transmission power $p_t(t)$ through a function Eq. (2) in a single user point-to-point transmission channel [16]–[20], [22]–[24].

$$r(t) = \log(1 + \frac{p_t(t)}{N}),^1 \tag{2}$$

where N is the average power of the AWGN channel noise and often assumed $N = 1$ [16]–[18], [22]–[24].

As a result, the total amount of data transmitted during $[0, t]$ can be calculated by the following integration,

$$B(t) = \int_0^t r(x)\,\mathrm{d}x \tag{3}$$

$$= \begin{cases} \sum_{j=1}^{i} \int_{t_{2j-1}}^{t_{2j}} r(x)\,\mathrm{d}x & t_{2i} \le t < t_{2i+1} \\ \sum_{j=1}^{i-1} \int_{t_{2j-1}}^{t_{2j}} r(x)\,\mathrm{d}x + \int_{t_{2i-1}}^{t} r(x)\,\mathrm{d}x & \\ & t_{2i-1} \le t < t_{2i} \end{cases} \tag{4}$$

Thus, the causality constraint can be expressed as

$$B(t) \le \sum_{i:a_i<t} B_i, \quad \forall t > 0. \tag{5}$$

According to Eq. (2) and (1), we have the transmission power as $p_t(t) = 2^{r(t)} - 1$. The total energy consumed during $[0, t]$ can be calculated by the following integration,

$$E(t) = \int_0^t p_t(x)\,\mathrm{d}x \tag{6}$$

$$= \begin{cases} \sum_{j=1}^{i} \int_{t_{2j-1}}^{t_{2j}} p_t(x)\,\mathrm{d}x & t_{2i} \le t < t_{2i+1} \\ \sum_{j=1}^{i-1} \int_{t_{2j-1}}^{t_{2j}} p_t(x)\,\mathrm{d}x + \int_{t_{2i-1}}^{t} p_t(x)\,\mathrm{d}x & \\ & t_{2i-1} \le t < t_{2i} \end{cases} \tag{7}$$

Suppose the battery capacity is E_b and the initial energy in battery is E_0. In any time instance t, the total energy consumed $E(t)$ can not exceed the received energy $H(t)$ plus the initial energy E_0 in the battery, this is called the *energy constraint*.

$$E_0 + H(t) - E(t) \ge 0, \quad \forall t \in [0, t_{2m}], \tag{8}$$

where $E_0 + H(t) - E(t)$ is also called the *remain energy* in the battery. Such a remain energy can not exceed the battery capacity.

$$E_0 + H(t) - E(t) \le E_b, \quad \forall t \in [0, t_{2m}], \tag{9}$$

Let $T = t_{2m}$ be the end of the last phase, then at T, all packets must have been completely transmitted. This is called *load constraint* expressed by the following equation,

$$B(T) = \sum_{i=1}^{n} B_i. \tag{10}$$

Time T is called the *transmission delay* or *completion time*.

The formal definition of the problem is given below.

Definition 2 (Delay Minimization Scheduling Problem, DMS Problem): Given a set of packets P and a wireless power transmission system described above, the delay minimization transmission scheduling problem is to determine the number of cycles m, all the switch points $t_1, t_2, \ldots t_{2m}$ and the transmission rate $r(t), 0 \le t \le T$ such that the causality constraint Eq. (5), the energy constraint Eq. (8), the battery capacity constraint Eq. (9) and the load constraint Eq. (10) are satisfied and the transmission delay T is minimized.

The DMS problem is called offline case if packet set P is completely known before scheduling. The transmission rate $r(t)$ in the offline optimal solution for this problem is denoted as $r^{opt}(t)$. It is called online problem if any packet P_i is not known until its arrival time a_i, $i = 1, 2, \ldots, n$. In other words, phase switching and transmission rate are determined based on the packet information before the current time.

IV. The *wOPT Rate*

This section concentrates on a simplified scenario where only one packet is in P and battery has sufficiently large capacity, namely DMS-1 problem. A surprising result is that, in the optimal solution that minimizes the completion time, the transmission rate depends on neither the packet size nor the initial energy when the initial energy is below a criterion (Theorem 1).

Suppose in the DMS-1 problem, the only packet arrives at time 0 and has a size B. Imagine that if no WPT is available, in order to minimize transmission delay, we can use a single rate throughout the transmission to delivery data. The correctness of such a single-rate transmission lies in the convex property of the power-rate function Eq. (2). More specifically, if two transmission rates are used, we can always find a single rate in between that delivers the same amount of data in a shorter time. Detailed proof can be found in [16]–[18] and [22]. Suppose such single rate is r, then the completion time $\tau = \frac{B}{r}$. All energy should be used up at the end, thus the transmission power is $p_t = \frac{E_0}{\tau} = \frac{E_0}{B} r$, where E_0 is the initial energy. Combining with Eq. (2), we have

$$\log(1 + \frac{E_0}{B} r) = r.$$

This equation can be solved to obtain the value r,

$$r = -\frac{\mathcal{W}\left(-\frac{B \ln 2}{E_0 2^{\frac{B}{E_0}}}\right)}{\ln 2} - \frac{B}{E_0}, \tag{11}$$

where function $\mathcal{W}(z)$ is called the Lambert W function [25], which has the following property,

$$\mathcal{W}(z) e^{\mathcal{W}(z)} = z.$$

Therefore the completion time can be computed as follows,

$$\tau = B/r = -B / \left(\frac{\mathcal{W}\left(-\frac{B \ln 2}{E_0 2^{\frac{B}{E_0}}}\right)}{\ln 2} + \frac{B}{E_0} \right). \tag{12}$$

We can see from Eq. (11) and (12) that both the transmission rate r and the completion time τ depend on B and E_0.

Now imagine the wireless device has another option that it can receive wireless power supply to charge the battery.

[1]Complex transmission is assumed here, and for the real transmission with power-rate function $r(t) = \frac{1}{2}\log(1 + \frac{p_t(t)}{N})$, all research results hold after scaling the transmission rate by $\frac{1}{2}$.

Since the battery is sufficiently large, it is easy to see that one *charging phase* and one *sending phase* is enough, *e.g.,* $m = 1$. This is because if $m > 1$, we can always combine all *charging phases* and all *sending phases* together without affecting the completion time. We assume the only *charging phase* is with length τ_1, and the only *sending phase* is with length τ_2. In order to minimize completion time, we must use a single transmission rate in the *sending phase*. Let it be r.

As a result, the total amount of energy in the battery by the end of the *charging phase* is $E_0 + p\tau_1$; the total amount of energy consumed in the *sending phase* is $\tau_2(2^r - 1)$. They must be equal,

$$E_0 + p\tau_1 = \tau_2(2^r - 1). \tag{13}$$

Since all data B is completely delivered, we have

$$\tau_2 = \frac{B}{r}. \tag{14}$$

As a result, Eq. (13) can be re-written as

$$E_0 + p\tau_1 = \frac{B}{r}(2^r - 1). \tag{15}$$

Multiplying p on both sides of Eq. (14), we get

$$p\tau_2 = p\frac{B}{r}. \tag{16}$$

Adding Eq. (15) to Eq. (16), we have

$$E_0 + p\tau_1 + p\tau_2 = \frac{B}{r}(2^r - 1 + p). \tag{17}$$

We further have

$$\tau_1 + \tau_2 = \frac{B(2^r - 1 + p)}{rp} - \frac{E_0}{p}. \tag{18}$$

Eq. (18) shows that the total completion time $T = \tau_1 + \tau_2$ is a function of variable r, which also depends on the data size B and the initial energy E_0.

We define the function $T(r)$ as

$$T(r) = \tau_1 + \tau_2 = \frac{B(2^r - 1 + p)}{rp} - \frac{E_0}{p} \tag{19}$$

Now, an interesting problem is to find the value of r such that the delay time $T(r)$ is minimized for a given initial energy E_0 and data size B. To find this value, let $T(r)' = (\frac{B(2^r-1+p)}{rp} - \frac{E_0}{p})' = 0$. We have

$$(\frac{2^r - 1 + p}{r})' = 0.$$

Solving this equation, we get,

$$r = \frac{\mathcal{W}(\frac{p-1}{e}) + 1}{\ln 2}. \tag{20}$$

Letting $w = \mathcal{W}(\frac{p-1}{e})$, we define r_s as follows

$$r_s = \frac{w + 1}{\ln 2}. \tag{21}$$

We called r_s in Eq (21) the *wOPT rate*, standing for *the optimal transmission rate by wirelessly powered transmitter*. When data is transmitted at the *wOPT rate* r_s, the delay time $T(r)$ is minimized, and the minimum value is $T(r_s)$. The *wOPT rate* r_s is a constant because it depends only on $w = \mathcal{W}(\frac{p-1}{e})$, which depends only on p and p is a constant.

Note that r_s depends on neither data size B nor battery initial energy E_0. However, the two phase lengths τ_1 and τ_2 depend on both. They can be calculated as follows.

$$\tau_2 = \frac{B}{r_s}, \quad \tau_1 = \frac{\frac{B}{r_s}(2^{r_s} - 1) - E_0}{p}, \quad \text{for } E_0 \leq \frac{B}{r_s}(2^{r_s} - 1), \tag{22}$$

where τ_1 is positive only if the initial energy E_0 is small, *i.e.,* $E_0 \leq \frac{B}{r_s}(2^{r_s} - 1)$.

If $E_0 > \frac{B}{r_s}(2^{r_s} - 1)$, energy in battery is already sufficient to send all B data, the WPT is unnecessary. Thus, the minimum delay is computed by Eq. (12).

We hence summarize and emphasize our conclusion of this section in Theorem 1, whose correctness follows directly from the above discussion.

Theorem 1: If the initial energy $E_0 \leq \frac{B}{r_s}(2^{r_s} - 1)$, then the optimal solution of the DMS-1 problem consists of a charging phase and a sending phase. The transmission rate in the sending phase is a constant r_s by (21) although the length of the two phases depend on E_0 and B, by (22). If $E_0 > \frac{B}{r_s}(2^{r_s} - 1)$, no charging phase is needed, thus the completion time can be computed by (12).

In fact, the notion *wOPT rate* is so important that not only it is the unique optimal rate to achieve the minimum delay, but it is also the optimal rate for the dual problem. The dual problem asks to maximize the remain energy when transmitting the packet before a given deadline. It is not difficult for readers to follow similar approaches discussed above to solve the dual problem. An example of using similar approaches to solve a related problem can be found in [26]. We omit details here, but will use this conclusion directly in later sections.

In later sections, we will show that although the *wOPT rate* is derived from the simple scenario, it indeed plays an important role in the general scenario. Therefore, we conclude that the discovery of the *wOPT rate* reveals an essential property of WPT.

V. The Offline Optimal Solutions

In this section, we study the offline DMS problem, where information about all packets in set P is known, including packet number, arrival time and sizes. We first investigate the problem where battery capacity E_b is sufficiently large and then solve the general problem by considering an arbitrary size battery capacity E_b.

A. An Optimal Solution for the Large Battery DMS Problem

In this subsection, we allow n packets in set P, $P = \{P_1, P_2, \ldots, P_n\}$ and $P_i(B_i, a_i)$, but still assume the battery capacity is sufficiently large. A large battery capacity allows us to combine all the *charging phases* together into one *charging phase* and does not affect the completion time. We thus focus on finding the optimal one (energy receiving, data transmitting) cycle solution.

If all packets arrive immediately after time 0, we can treat them as a single packet with size $B = \sum_{1\leq i\leq n} B_i$ and the *wOPT rate* r_s is still the optimal rate in the *sending phase* as discussed in the last section. However, if some packets arrive very late, then at some time point t, the transmitter is forced to stop because all arrived data have been transmitted and some packets in P have not arrived yet. We could charge more energy while we are waiting for these packets to arrive. This extra energy allows us to use a higher rate than r_s to shorten the completion time.

Before we present the optimal algorithm that produces the minimum completion time T, we would like to state some properties of the optimal solution, *i.e.,* the optimal rate schedule, should have. Since we focus on the one cycle solution, the following lemmas are about the transmission rate in the only *sending phase*.

Lemma 1: The optimal rate schedule $r^{opt}(t)$ is a non-decreasing function until the completion time T.

Lemma 2: The optimal rate schedule $r^{opt}(t)$ increases only at a packet arrival time $a_i, 1 \leq i \leq n$.

Lemma 3: The optimal rate schedule $r^{opt}(t)$ increases only when all arrived data has been transmitted.

Lemmas similar to Lemmas 1, 2 and 3 have been known in the literature for energy efficient wireless transmissions [17], [18] and energy harvesting wireless transmissions [16], [22], [34]. We therefore omit formal proofs, but provide some intuitive interpretations and explanations. In any rate schedule $r(t)$, two rates (in two durations) can be equalized to a singe rate (for both durations) consuming less energy, and therefore shorter transmission delay. This method is called *equalization*, whose correctness is behind the convex property of the power-rate function. All three lemmas can be proved by contradictions with the *equalization* method. Interested readers are suggested to refer to [34] for more details.

We now introduce the *cumulative data-time* diagram [20]. Let $A(t) = \sum_{i:a_i \leq t} B_i$ denote the total amount of bits that have arrived in time interval $[0, t]$. The curve of function $A(t)$ on the *cumulative data-time* diagram is called the *arrival curve*. Obviously, the *arrival curve* is with an up-stair-like shape, as depicted in Fig. 3. Similarly, we define the departure curve $B(t)$, which is the actual amount of data leaving the system (transmitted) during $[0, t]$. It is easy to see that a feasible departure curve $B(t)$ must be on the right side of the arrival curve $A(t)$ because of the causality constraint. Furthermore, the slope of $B(t)$ is actually the rate schedule $r(t)$. Let $B^{opt}(t)$ be the optimal departure curve, then, the determination of $B^{opt}(t)$ immediately leads to the determination of $r^{opt}(t)$. We therefore focus on finding $B^{opt}(t)$.

The high level idea to solve the large battery DMS problem is as follows. In the *cumulative data-time* diagram, *i.e.,* Fig. 3, we draw a line segment with slope r_s connecting point $(\tau_1, 0)$ and point $(\tau_1 + \tau_2, B)$, where τ_1 and τ_2 are calculated by Eq. (22) and $B = \sum_{1 \leq i \leq n} B_i$. As in Fig. 3, such line segment is $L'K'$, which represents the optimal rate schedule to transmit an amount of B data. If line segment $L'K'$ is on the right of the arrival curve $A(t)$, then such rate schedule is feasible and minimizes the completion time, thus we are done. Otherwise, we move this line segment towards its right, stop moving as soon as it is on the right of the arrival curve, *i.e.,* LK in Fig. 3.

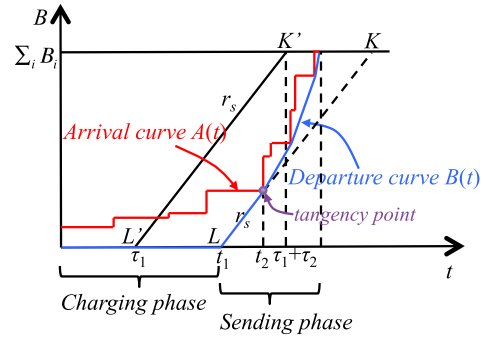


Fig. 3. In the *cumulative data-time* diagram, a feasible departure curve must be on the right side of the arrival curve. The slopes of the departure curve is the transmission rates. The optimal departure curve for the corresponding DMS-1 problem can be represented by the line segment $L'K'$. If $L'K'$ is on the right side of the arrival curve, we are done. Otherwise, we move $L'K'$ right to a new position LK such that the arrival curve is on its left side and there is a tangency point. From this tangency, we iteratively find the optimal line segment.

Obviously, there is a tangency point on LK. The rightward movement suggests that the transmission starts at a later time $t_1, t_1 > \tau_1$, such that we get more energy charged into the battery and therefore can use a higher-slope line segment to minimize the completion time. We take this tangency point as the start point, take the remain energy in battery as E_0 and take the total amount of unsent packets as B, then we compute r_{nowpt} by Eq. (11) to minimize the completion time. If the line starting from this tangency point with slope r_{nowpt} is on the right of the arrival curve, we are done. Otherwise, we connect this tangency point and every corner of the arrival curve to find the lowest-slope line segment. We now take the ending point of this line segment as a start point and repeat this process until all packets are finished.

We present formal steps of this method in Algorithm DMSP-LargeBattery. Line 3-6 test whether line segment $L'K'$ is on the right of $A(t)$. Line 7-9 directly compute the position of LK. The **while** loop repeatedly computes r_{nowpt} and the lowest-slope line segment.

Theorem 2: Algorithm DMSP-LargeBattery *produces the optimal departure curve $B^{opt}(t)$ for the offline DMS problem with a sufficiently large battery within $O(n^2)$ steps.*

Proof: See Appendix A. □

B. An Optimal Solution for the General Problem

This subsection studies the original DMS problem of Definition 2, in which the battery has a capacity of E_b. Unlike the solution in the previous subsection, multiple (energy receiving, data transmitting) cycles are needed in this general case.

We will show how to divide the time into cycles and how to determine the transmission rate for each cycle, but first we introduce one property about the optimal rate schedule.

Theorem 3: In an optimal rate schedule $r^{opt}(t)$, the rate in every cycle is r_s except the one in the last cycle.

Proof: Appendix B. □

In the last cycle, Lemmas 1, 2 and 3 still hold. Obviously, in the last cycle of the optimal solution, for two operations, *i.e.,* the energy harvesting in the charging phase and the data

Algorithm 1 DMSP-LargeBattery

Set $B_0 = 0$ for loop purpose;
Let τ_1 and τ_2 be calculated by Eq. (22);
$t_1 = \max_i(a_i - \frac{\sum_{j=0}^{i-1} B_j}{r_s})$;
if $t_1 < \tau_1$ **then**
  return line segment $(\tau_1, 0) - (\tau_1 + \tau_2, B)$;
end
$k = \arg\max_i(a_i - \frac{\sum_{j=0}^{i-1} B_j}{r_s})$;
$t_2 = a_k$;
Set line segment $(t_1, 0) - (t_2, \sum_{j=0}^{k-1} B_j))$;
while $k < n$ **do**
  $r_{min} = \min_{k<i\leq n} \frac{\sum_{j=k}^{i-1} B_j}{a_i - a_k}$;
  Take the remain energy in battery as E_0 and take $\sum_{k\leq i\leq n} B_i$ as B, compute r_{nowpt} by Eq. (11);
  if $r_{nowpt} < r_{min}$ **then**
    Set segment $(a_k, \sum_{j=0}^{k-1} B_i) - (a_k + \frac{\sum_{k\leq i\leq n} B_i}{r_{nowpt}}, B)$;
    return all line segments;
  else
    $k_{new} = \arg\min_{k<i\leq n} \frac{\sum_{j=k}^{i-1} B_j}{a_i - a_k}$; Set segment $(a_k, \sum_{j=0}^{k-1} B_i) - (a_{k_{new}}, \sum_{j=0}^{k_{new}-1} B_i)$;
    $k = k_{new}$;
  end
end
Take the remain energy as E_0 and take B_n as B, compute r_{nowpt} by Eq. (11).;
Set line segment $(a_n, B - B_n) - (a_n + \frac{B_n}{r_{nowpt}}, B)$;
return all line segments;

transmitting in the sending phase, the battery is large enough, because otherwise it should not be the last cycle. Hence, to transmit these data, algorithm DMSP-LargeBattery can be used to compute the transmission schedule. Although we have known the transmission rate in each cycle, we still need to determine the beginning and ending of cycles.

In this section, we assume the battery is initially empty, *i.e.*, $E_0 = 0$. Any non-zero initial energy $E_0 \neq 0$ can be equivalently considered as if an empty battery being charged for a length of E_0/p time. We therefore move the starting time earlier, and during the first E_0/p time, no packet arrives such that charging battery is the only option, and by original starting time, there is E_0 energy in the battery.

In every cycle except the last one, the transmission rate is r_s in *sending phases* according to Theorem 3. To support a τ_2'-length *sending phase* which transmits $B' = r_s\tau_2'$ data, we need a τ_1'-length *charging phase* to accumulate energy where $\tau_1' = \frac{(2^{r_s}-1)\tau_2}{p}$, In other words, any amount of B' data takes at least $\tau_1' + \tau_2' = \frac{2^{r_s}-1+p}{p} \times \frac{B'}{r_s}$ time to transmit. We define the *effective transmission rate* $r_a = \frac{B'}{\tau_1'+\tau_2'} = \frac{r_s p}{2^{r_s}-1+p}$, which is independent of data amount B'.

The following lemma discusses an important property of the multiple-cycle solution.

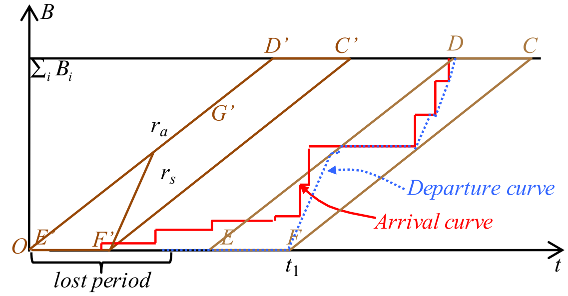


Fig. 4. From point O, system starts with an empty battery. In parallelogram $C'D'E'F'$ with $E' = O$, $E'F'$ is with length $\frac{E_b}{p}$, which is the time duration to charge the battery to full. Both $D'E'$ and $C'F'$ are with slope r_a, which is the maximum effective transmission rate. If no energy overflow is allowed, then the departure curve will not leave $C'D'E'F'$. By collection all lost periods (charging periods that cause overflow) to put at the beginning, such parallelogram can be delayed. We hence move $C'D'E'F'$ rightwards to a new position $CDEF$ such that the arrival curve is on the left side of CF. The length OE is the lost period which does not contribute to the energy in battery, so battery is empty at point E. The optimal departure curve can be re-arranged to be bounded by $CDEF$ since no energy overflow after point E. We then repeat charging and sending until the unsent data is small, then we invoke the optimal algorithm for the large battery case to complete the transmission.

Lemma 4: On the cumulative data-time *diagram, any feasible departure curve $B(t)$, including the optimal $B^{opt}(t)$, can be re-arranged to be bounded by a parallelogram with bottom length E_b/p, side line slope r_a and height $B = \sum_i B_i$.*

Two example parallelograms are $C'D'E'F'$ and $CDEF$ in Fig. 4, where the bottom edges $E'F'$ and EF are with length E_b/p, and the side edges $D'E'$, $C'F'$, DE and CF are all with slope r_a, and both heights are B.

Proof: In a feasible solution, it is likely some energy is wasted because of the battery capacity constraint. In case the battery is already full, any further energy harvested will be lost. The charging period when the battery is full is called *the lost period*, which does not contribute to the energy in battery. Such lost periods may be in many *charging phases*. We modify the solution to combine all the lost periods into a single one and put it at the beginning of time. Obviously, the modified solution is still feasible and the transmission delay is not affected.

Suppose $t \geq 0$ is the ending of the lost period, then at time t, the battery is empty since lost periods do not contribute to energy in battery, and after t, no energy is wasted. Let the parallelogram starts at point $(t, 0)$, then the modified feasible departure curve will not leave such parallelogram. This is because, whenever the departure curve reaches the right boundary by charging battery, the battery is full and it must switch to sending data since no energy should be wasted, therefore the curve goes up right. Whenever the departure curve reaches the left boundary by a sending phase, the battery becomes empty and the transmitter must stop sending and switch to charging, so the departure curve horizontally goes to the right. As a conclusion, the modified feasible departure curve must be inside $CDEF$. □

Although we have known that the optimal departure curve (and any feasible departure curve) must be inside the parallelogram, its starting time t is unknown yet. We hence first determine the optimal position of such parallelogram, then determine the optimal departure curve inside this parallelogram. An example of the optimal departure curve inside the optimal parallelogram is given in Fig. 4 to provide readers the perceptual intuition.

The high level idea is as follows. When the parallelogram is at its leftmost position, i.e., $C'D'E'F'$ where $E' = O$, if $C'F'$ is already on the right side of the arrival curve, then, $t = 0$ is the optimal position. Otherwise, the parallelogram is moved rightwards to a new position $CDEF$, at which CF is on the right of the arrival curve and they share one common point. We will prove later that this is the optimal position. Therefore, we are safe to charge until the position of F, then, we follow a simple strategy: we start a sending phase to transmit at the *wOPT rate* until no energy or data left, then switch to charging phase until battery full or a packet arrival. We repeat the (energy receiving, data transmitting) cycle until the total unsent data drops to a small value such that the battery capacity seems sufficiently large for transmitting them. Then we call previously designed algorithm for large battery case to complete the transmission schedule. The detailed pseudo code is presented in Algorithm DMSP.

Algorithm 2 DMSP

1 Set $r_s = \frac{w+1}{\ln 2}$ `// the wOPT rate`
2 Set $r_a = \frac{r_s p}{2^{r_s}-1+p}$;
3 Set $t_0 = \max\{\frac{E_b}{p}, \max_i\{a_i - \frac{\sum_{j=0}^{i-1} B_j}{r_a}\}\}$;
4 Charge until time t_0;
5 Set $B = \sum_{i=1}^{n} B_i$ and $B_{sent} = 0$;
6 **while** $B - B_{sent} > \frac{E_b r_s}{2^{r_s}-1}$ **do**
7   **if** *no backlog data* **then**
8   Charge until a packet arrival;
9   **else if** *no remain energy* **then**
10   Charge until battery full;
11   Transmit at r_s until a packet arrival or no energy/data left;
12   Update the B_{sent} by the transmitted amount ;
13 **end**
14 Invoke DMSP-LargeBattery;

Observation 1: Any energy overflow is caused by charging phase set in Line 8, e.g., *before a late packet arrival.*

Theorem 4: Algorithm DMSP *produces the optimal rate schedule for the general offline DMS problem within* $O(\min\{\frac{B\times(2^{r_s}-1)}{r_s E_b}, n^2\})$ *steps.*

Proof: See Appendix C. □

VI. The Online Algorithm

Based on the *wOPT rate* discovered from the offline analysis, this section develops an online transmission delay minimization scheduling algorithm without any knowledge of the distributions of packet size and arrival time. We prove that the proposed algorithm is 1.16-competitive in case the battery is initially empty.

Algorithm 3 DMSP-Online

1 Set $r_s = \frac{w+1}{\ln 2}$ `// the wOPT rate`
2 Set remain data $B_{rem} = 0$ and remain energy $E_{rem} = 0$;
3 **while** *true* **do**
4   **if** $B_{rem} == 0$ **then**
5   Charge until a packet arrival;
6   **else if** $E_{rem} == 0$ **then**
7   Charge for $\min\{\frac{B_{rem}(2^{r_s}-1)}{pr_s}, \frac{E_b - E_{rem}}{p}\}$ time;
8   **end**
9   Transmit at r_s until no energy or data left;
10   Update remain data B_{rem} and remain energy E_{rem};
11 **end**

A. The Algorithm Design

Since the *wOPT rate* has already been proved to be optimal for the offline algorithm, we use it as the only transmission rate in the online case. Although we do not have the information on packet sizes and arrival times, the online algorithm uses *wOPT rate* to deliver data whenever both data and energy are ready, in a way quite similar to the offline optimal Algorithm DMSP. Details are given in DMSP-Online.

We can see that each **while** loop iteration is one (energy receiving, data transmitting) cycle. For the *charging phase* length, there are two cases. First, if there is no data left, then the *charging phase* lasts until the next packet arrival, as in Line 5. Second, if there is no energy remain but some data left, then the charging lasts until either energy is enough to transmit these data *e.g.,* $B_{rem}(2^{r_s} - 1)/(p \cdot r_s)$, or the battery is full *e.g.,* $(E_b - E_{rem})/p$, as shown in Line 7. The *sending phase* lasts until either no energy or no data left, as in Line 9.

B. Competitive Analysis

The competitive analysis is to measure the worst-case performance of online algorithms. Let $ALG(\sigma)$ denote the transmission delay by the proposed DMSP-Online algorithm for the input packet set instance σ, while $OPT(\sigma)$ denotes the offline optimal transmission delay with complete knowledge on σ, including the packet number, packet sizes and arrival times. We say the online algorithm is λ-competitive if it always produces a transmission delay within λ times of the offline optimal value for any input σ. That is,

$$\max_{\sigma} \frac{ALG(\sigma)}{OPT(\sigma)} \leq \lambda.$$

We start our competitive analysis with the large battery case. The transmission delays by online algorithm DMSP-Online and the offline optimal algorithm DMSP-LargeBattery are denoted as $ALG_l(\sigma)$ and $OPT_l(\sigma)$, respectively. They are also referred as ALG_l and OPT_l for short. Then, we want to prove

$$\max_{\sigma} \frac{ALG_l(\sigma)}{OPT_l(\sigma)} < 1.16.$$

In other words, our online algorithm is 1.16-competitive if the battery is sufficiently large.

Since the offline solution by DMSP-LargeBattery has one cycle, *i.e.,* $OPT_l = \tau_1 + \tau_2$, where τ_1 and τ_2 are charging and sending phase lengths respectively. Although the sending phase τ_2 varies with different input σ, it has a lower bound τ_2^{lb}, which is the sending phase length if a single transmission rate is used to deliver all B data consuming all energy charged. Then, we have

$$\tau_2^{lb} \log(1 + \frac{p\tau_1}{\tau_2^{lb}}) = B, \tag{23}$$

from which we can see the lower bound τ_2^{lb} depends on τ_1 and B. When B is given, the larger τ_1 the smaller τ_2^{lb} and the smaller τ_1 the larger τ_2^{lb}. We use $\tau_1 + \tau_2^{lb}$ as a lower bound for $OPT_l(\sigma)$.

For any given instance σ, we modify our online solution by combining charging phases and sending phases respectively into one cycle. In the combined sending phase, r_s is the only transmission rate according to the online algorithm. After modification, the transmission delay ALG_l keeps the same since the battery is large and there is no energy overflow after the combination.

Lemma 5: The online and offline solution share the same charging phase length τ_1 and $\tau_1 \geq \tau_1^s$, where $\tau_1^s = \frac{B(2^{r_s}-1)}{r_s p}$.

Proof: See Appendix D. □

Since r_s is the only transmission rate in the combined sending phase, its length is $\tau_2^s = B/r_s$. Obvious, the online transmission delay $ALG_l(\sigma) = \tau_1 + \tau_2^s$.

The notation of τ_1, τ_2, τ_1^s, τ_2^s, τ_2^{lb}, the arrival curve $A(t)$, the modified departure curve $B^{online}(t)$ by DMSP-Online and the offline optimal departure curve $B^{offline}(t)$ by DMSP-LargeBattery are depicted on *cumulative data-time* diagram in Fig. 5.

The competitive ratio between the two delays can be computed as

$$\frac{ALG_l}{OPT_l} = \frac{\tau_1 + \tau_2^s}{\tau_1 + \tau_2} \leq \frac{\tau_1 + \tau_2^s}{\tau_1 + \tau_2^{lb}},$$

which is determined by τ_1. When τ_1 is at its smallest value, $\tau_1 = \tau_1^s$, then $\tau_2^{lb} = \tau_2^s$ and the competitive ratio is also minimized at 1. When τ_1 grows, the ratio grows too, because the gap between t_2^{lb} and τ_2^s enlarged. However, when τ_1 grow large enough, it dominates both the numerator and denominator in ratio computation, so the ratio drops when τ_1 grows even larger. This observation indicates the competitive ratio is bounded, and inspires our theoretical analysis.

From (23) we have

$$p\tau_1 = (2^{B/\tau_2^{lb}} - 1)\tau_2^{lb},$$
$$\tau_1 = \frac{(2^r - 1)B}{rp}, \tag{24}$$
$$\tau_1 + \tau_2^{lb} = \frac{(2^r - 1 + p)B}{rp}. \tag{25}$$

By $\tau_2^s = B/r_s$, (24) and (25), we have

$$\frac{\tau_1 + \tau_2^s}{\tau_1 + \tau_2^{lb}} = \frac{2^r - 1 + \frac{r}{r_s}p}{2^r - 1 + p}.$$

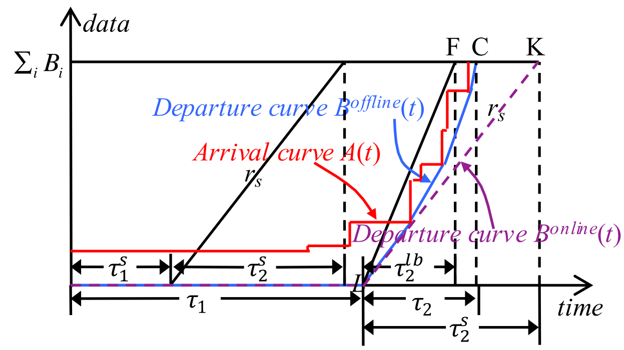


Fig. 5. Competitive ratio analysis. The offline solution by DMSP-LargeBattery has one cycle, where τ_1 (τ_2) is charging (sending) phase length. τ_2 has a lower bound τ_2^{lb} (reach when send at a constant rate), and τ_2^{lb} depends on τ_1, *i.e.,* the longer (shorter) time charging, the shorter (longer) sending time lower bound. When battery is large, we can combine online solution by DMSP-Online into one cycle. Its charging phase length is also τ_1. We must have $\tau_1 \geq \tau_1^s$, where τ_1^s is the charging phase length that supports transmit B data at rate r_s for $\tau_2^s = B/r_s$ time. Obviously, the online transmission delay is $\tau_1 + \tau_2^s$. Then, the competitive ratio between the two delays should be bounded by $\frac{\tau_1+\tau_2^s}{\tau_1+\tau_2^{lb}}$, which is determined by τ_1. When τ_1 is at its smallest value, $\tau_1 = \tau_1^s$, the competitive ratio is also minimized at 1. When τ_1 grows, the ratio grows too, because the gap between t_2^{lb} and τ_2^s enlarged. However, when τ_1 grow large enough, it dominates both the numerator and denominator in ratio computation, so the ratio drops when τ_1 grows even larger. This observation indicates the competitive ratio is bounded, and inspires our theoretical analysis.

Therefore, it is a function of p and r. Let

$$f(p, r) = \frac{2^r - 1 + \frac{r}{r_s}p}{2^r - 1 + p}.$$

To get the maximum value of $f(p, r)$, we must have

$$\frac{\partial f}{\partial r} = 0.$$

Since $r_s = \frac{w+1}{\ln 2}$ and $w = \mathcal{W}(\frac{p-1}{e})$, so

$$2^r + \frac{2^r - 1 + p}{w+1} = \frac{r 2^r \ln 2}{w+1}$$
$$(r\ln 2 - w - 2)2^r = p - 1$$
$$\mathcal{W}(\frac{p-1}{e^{w+2}}) = r\ln 2 - (w+2) \tag{26}$$
$$\mathcal{W}(\frac{w}{e}) = r\ln 2 - (w+2) \tag{27}$$
$$r = \frac{\mathcal{W}(\frac{w}{e}) + w + 2}{\ln 2}. \tag{28}$$

Note that the change from (26) to (27) is because we have

$$\frac{p-1}{e^{w+2}} = \frac{1}{e^{w+1}}\frac{p-1}{e} = \frac{1}{e^{w+1}}\mathcal{W}(\frac{p-1}{e})e^{\mathcal{W}(\frac{p-1}{e})} = \frac{w}{e}.$$

Furthermore, we have

$$\begin{aligned} 2^r &= e^{r\ln 2} \overset{(a)}{=} e^{\mathcal{W}(\frac{w}{e})}e^{w+2} \\ &= \frac{\mathcal{W}(\frac{w}{e})e^{\mathcal{W}(\frac{w}{e})}}{\mathcal{W}(\frac{w}{e})}e^{w+2} = \frac{\frac{w}{e}}{\mathcal{W}(\frac{w}{e})} \times \frac{p-1}{\frac{p-1}{e^{w+2}}} \\ &= \frac{w}{e\mathcal{W}(\frac{w}{e})} \times \frac{p-1}{\frac{w}{e}} = \frac{p-1}{\mathcal{W}(\frac{w}{e})}, \end{aligned}$$

where equation (a) is because of (28).

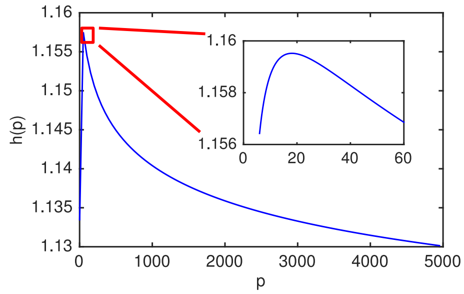


Fig. 6. The shape of function $h(p)$ showing the trends and maximum value.

Assume point (p^*, r^*) is the one where $f(p,r)$ achieves the maximum value, then they must satisfy the following equations.

$$r_s^* = \frac{\mathcal{W}(\frac{p^*-1}{e}) + 1}{\ln 2}$$
$$r^* = \frac{\mathcal{W}(\frac{w^*}{e}) + w^* + 2}{\ln 2}$$
$$2^{r^*} = \frac{p^* - 1}{\mathcal{W}(\frac{w^*}{e})}.$$

We now pose $f(p^*, r^*)$ as

$$f(p^*, r^*) = \frac{2^{r^*} - 1 + \frac{r^*}{r_s^*}p^*}{2^{r^*} - 1 + p^*} = 1 + \frac{p^*\mathcal{W}(\frac{\mathcal{W}(\frac{p^*-1}{e})}{e})}{(p^*-1)(\mathcal{W}(\frac{p^*-1}{e}) + 1)}.$$

Although we do not know the exact value of p^*, we define a function $h(p)$:

$$h(p) = 1 + \frac{\mathcal{W}(\frac{\mathcal{W}(\frac{p-1}{e})}{e})p}{(p-1)(\mathcal{W}(\frac{p-1}{e} + 1)}.$$

So,

$$f(p^*, r^*) \le \max_p h(p).$$

See Fig. 6 for the shape of function $h(p)$. Therefore, when $p = 18.103060$, $h(p)$ has the largest value.

$$\max_p h(p) = h(18.103060) = 1.159521.$$

Finally,

$$\begin{aligned}\max_\sigma \frac{ALG_l(\sigma)}{OPT_l(\sigma)} &\le \max \frac{2^r - 1 + \frac{r}{r_s}p}{2^r - 1 + p} \\ &= \max f(p,r) = f(p^*, r^*) \\ &\le \max_p h(p) = h(18.103060) \\ &= 1.159521 < 1.16.\end{aligned}$$

In the general battery capacity case, online algorithm DMSP-ONLINE uses r_s in all transmission while the offline optimal DMSP used rate r_s until the transmitted data $B_{sent} > B - \frac{E_b r_s}{2^{r_s}-1}$. After that, DMSP-LARGEBATTERY is invoked. In other words, both DMSP-ONLINE and DMSP deliver the first B_{sent} data by rate r_s, thus they use the same amount of sending time and consume the same amount of energy, hence the charging time to harvest these energy is the same as well. Assume T_{rs} is the time for both algorithms to charging and sending for the first B_{sent} data, we subtract such time from the total time ALG and OPT respectively. Let $ALG_b = ALG - T_{rs}$ and $OPT_b = OPT - T_{rs}$, which is used to deliver the rest $B - B_{sent}$ data. Since the rest data $B - B_{sent}$ can be transmitted at rate r_s in one charging cycle even considering battery capacity E_b, *e.g.,* the battery is sufficient large to transmit the rest data. Denote ALG_l and OPT_l as the transmission delay for transmitting the same amount of $B - B_{sent}$ data assuming a sufficiently large battery. Obviously, we have $OPT_b > OPT_l$ and $ALG_b = ALG_l$. Then

$$\frac{ALG}{OPT} = \frac{T_{rs} + ALG_b}{T_{rs} + OPT_b} \overset{(c)}{\le} \frac{ALG_b}{OPT_b} \le \frac{ALG_l}{OPT_l},$$

where (c) is because if we have positive real numbers a, b, c and $a \ge b$, then $a/b \ge (a+c)/(b+c)$. So,

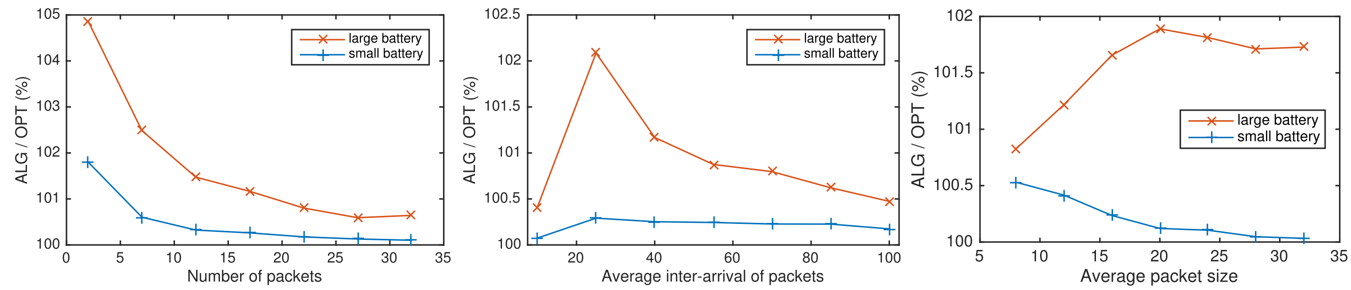
$$\max_\sigma \frac{ALG(\sigma)}{OPT(\sigma)} \le \max_\sigma \frac{ALG_l(\sigma)}{OPT_l(\sigma)} < 1.16.$$

VII. SIMULATIONS

In this section, we implement the proposed Algorithm DMSP-ONLINE and study its efficiency. Since there is no other existing algorithm studies the same DMS problem for a wireless powered device in the literature, we compare the online algorithm against the offline optimal Algorithm DMSP. We also use a simple heuristic derived from Yang and Ulukus [16] as a benchmark scheme. The simulation has two goals, one is to verify the theoretical result that the performance of DMSP-ONLINE is bounded by a constant 1.16 when the battery is initially empty, and the other goal is to evaluate its performance when the battery is with arbitrary initial energy.

Following Yang and Ulukus [16], we consider a band-limited additive white Gaussian noise channel, with bandwidth W = 1MHz and the noise power spectral density $N_0 = 10^{-19}$W/Hz. We assume that the distance between the transmitter and the receiver is 1km, and the path loss h is about 90dB. Then, we have $r = W\log_2(1 + \frac{ph}{N_0\ W}) = \log_2(1+p)$, where p is in milliwatts (mW) and r is in megabits (mb) per second. In simulations, n packets are generated following the Poisson arrival, where the average inter-arrival is set to be l s. Packet size is assumed to be a random variable following the uniform distribution with the average size b mb. The battery capacity is set to be 30 mJ and the energy transfer speed is 3 mW. For the non-empty initial battery case, the initial energy is assumed to be a random value between empty and full. Parameters n, l and b will be changed one at a time to evaluate their impacts on algorithm performance. For every settings in our simulation, we randomly generate 100 instances of packet set, and use the mean value of the results for comparisons.

The simulation results when battery is initially empty are presented in Fig. 7 and Table. I. Algorithm performance is evaluated by schedule transmission delay, which is denoted as ALG, OPT, ALG_l and OPT_l for the online algorithm



(a) (b) (c)

Fig. 7. The ratio between the transmission delays of the online scheme to that of the offline scheme for empty initial battery case. The default setting is: number of packets $n = 12$, average inter-arrival $l = 35$ seconds (s), average packet size $b = 14$ megabits (mb). The three parameters are changed one at a time as in sub-figures. Transmission delays are given in Table I.

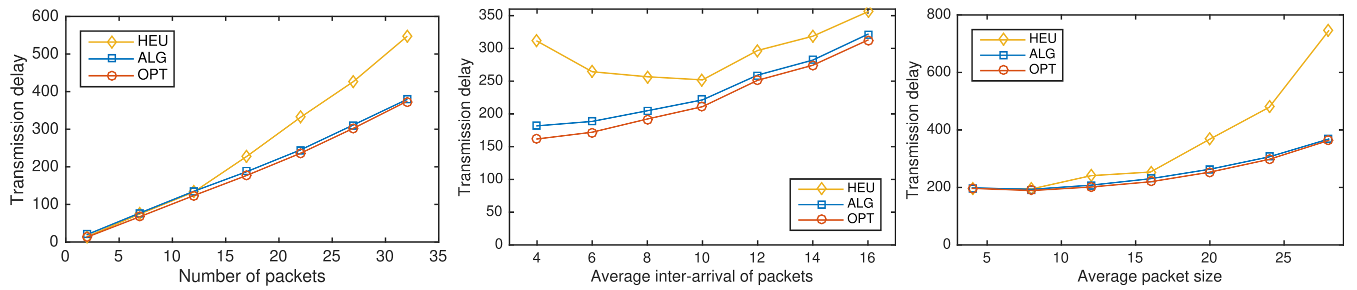
TABLE I
TRANSMISSION DELAYS OF FIG. 7

# packet	2	7	12	17	22	27	32
$ALG(\times 10)$	4.7	22.3	39.9	55.7	76.2	94.7	111.8
$OPT(\times 10)$	4.6	22.2	39.7	55.5	76.1	94.6	111.6
$ALG_l(\times 10)$	4.6	21.9	39.5	55.2	75.9	94.3	111.3
$OPT_l(\times 10)$	4.4	21.4	38.9	54.5	75.3	93.7	110.5
inter-arrival (s)	10	25	40	55	70	85	100
$ALG(\times 10)$	17.0	29.9	45.3	62.8	75.1	91.0	111.6
$OPT(\times 10)$	16.9	29.8	45.1	62.6	74.9	90.8	111.4
$ALG_l(\times 10)$	16.4	29.2	45.0	62.5	74.8	90.8	111.5
$OPT_l(\times 10)$	16.3	28.6	44.4	62.0	74.2	90.2	110.9
packet size (mb)	8	12	16	20	24	28	32
$ALG(\times 10)$	39.0	40.0	40.5	41.0	41.7	46.0	48.0
$OPT(\times 10)$	38.8	39.8	40.4	41.0	41.7	46.0	48.0
$ALG_l(\times 10)$	38.9	39.7	39.9	39.6	40.0	43.4	44.7
$OPT_l(\times 10)$	38.6	39.2	39.2	38.9	39.3	42.7	43.9

DMSP-ONLINE with 30 mJ battery, the offline optimal algorithm DMSP with 30 mJ battery, the online algorithm DMSP-ONLINE with large battery and the offline optimal algorithm DMSP with large battery respectively. The values of ALG, OPT, ALG_l and OPT_l are given in Table. I, while the small battery ALG-by-OPT ratio and large battery ALG_l-by-OPT_l ratio of three simulation scenarios are depicted in Fig. 7. We can see from Fig. 7(a) that the more packets, the better our online algorithm performances in terms of the ALG-by-OPT ratio. This is not hard to explain: since the average inter-arrival of packets does not change, more packets means longer time for all packets to arrive, hence the transmission completion time increases, which can be verified in Table. I. As a result, ALG-by-OPT ratio drops even as the gap between the two increases slightly. Fig. 7(b) shows that, with the increase of average packet inter-arrival, the ratio first grows and then drops. This is because when the inter-arrival is small, packets arrive shortly after beginning. The offline optimal algorithm is likely to transmission at *wOPT rate* even in the last cycle since packets are all ready and no extra energy charged to speed up. While our online algorithm always transmits at the *wOPT rate*. As the inter-arrival increases, more packets arrive lately, hence more energy is harvested by charging while waiting. Therefore higher rate can be used to shorten the delay. As a result, the ratio increases. However, when the inter-arrival grows even larger, the transmission completion time for both solution grows, as in Table. I, then the ratio drops as a result. In (a) and (b) the large battery ratios have the same trend with small battery ratios. However, in (c), the two have different trends. The large battery ratio first increases and then decreases as the average packet size grows. This is because, when packets are small, each packet is transmitted shortly after its arrival, both online and offline algorithm can achieve similar performance. As the size grows, more time is spent on sending. Because our online algorithm uses only the *wOPT rate*, hence the ratio grows. However, when the size grows even larger, the offline algorithm is more likely to use the *wOPT rate* in the last cycle because less energy remain in the battery, therefore the ratio drops. The small battery ratio decreases as the average packet size grows, because the more data to send, the more likely *wOPT rate* is used in the last cycle.

From all three sub-figures, we can see that our online algorithm performance is less than 1.05 of the offline optimal performance, far better than the theoretical 1.16 bound. From (a), we can see that the less packet number, the larger ratio. A practical system normally has more packets than two, thus the ratio is usually small. In summary, the simulation results verify the theoretical 1.16 bound on our online algorithm.

The simulation results when battery is with arbitrary initial energy are presented in Fig. 8 and Table. II. We use a simple heuristic HEU derived from Yang and Ulukus [16] as a benchmark scheme when battery is sufficiently large. When no energy harvesting or WPT is considered, their scheduling scheme reduces to the lazy scheduling [18], which is essentially same with steps in **while** of Algorithm DMSP-LARGEBATTERY. Now the problem is how to determine the length of the charging phase. We randomly choose one in our simple heuristic scheme HEU. In Fig. 8, performances of the heuristic scheme, the online and offline algorithm for a very large battery, *i.e.,* 1000 mJ, are compared. In Table. II, the ALG-by-OPT ratios are presented. We can see from Fig. 8(a) that the less packets, the more gap between the two curves ALG and OPT. This is because the offline optimal algorithm uses higher rates to shorten transmission delay if few packets are in consideration and the initial energy in battery is sufficient. Fig. 8(b) shows that the gap between the two curves ALG and OPT decrease as the average inter-arrival of packets increase. This is because a short inter-arrival means packets



(a) (b) (c)

Fig. 8. The performance comparison by transmission delay for random initial energy case. The default setting is: number of packets $n = 20$, average inter-arrival $l = 10$ seconds (s), average packet size $b = 16$ megabits (mb). The three parameters are changed one at a time as in sub-figures. The performance comparison ratios are given in Table II.

TABLE II
ONLINE TO OFFLINE RATIO OF FIG. 8

# packet	2	7	12	17	22	27	32
$\frac{ALG}{OPT}\%$	151.0	114.5	109.2	105.6	103.8	103.0	101.3
interval (s)	4	6	8	10	12	14	16
$\frac{ALG}{OPT}\%$	112.4	109.6	106.6	104.9	103.2	102.9	102.9
size (mb)	4	8	12	16	20	24	28
$\frac{ALG}{OPT}\%$	100.9	102.2	103.1	104.8	104.0	102.9	101.3

arrive the system shortly after begin. Then, in the optimal solution, the initial energy is more likely to support a fast transmission since packets are ready. In Fig. 8(c), as the average packet size increases, the performance gap first increases and then decreases. This is because, when packets are small, few time is needed to transmit, both online and offline algorithms can achieve similar performance. While, if packets are all large, a lot energy is consumed by transmission, it is less likely that there is extra energy to support higher rate in the last cycle to shorten delay. Not surprisingly, in (a), (b) and (c), the performance of the simple heuristic HEU is much worse than ALG and OPT. We can see from Table II that the ALG-by-OPT ratio is small in all three simulation scenarios, i.e., less than 151%, indicting the performance of our online algorithm is efficient. Only when the packet number is very small, namely 2, and the inter-arrival of packets is very small, namely 4, the online algorithm suffers worse performance compared to the offline optimal.

As a conclusion from these figures and tables, the online algorithm performance is near optimal, because our online algorithm is designed based on the discovery of the *wOPT rate*.

VIII. CONCLUSIONS

In this paper, we have studied the DMS problem for a WPT device. We designed an offline optimal schedule such that a sequence of data packets can be transmitted with the minimum delay assuming the point-to-point AWGN channel. It was discovered that, for all (energy receiving, data transmitting) cycles, except the last cycle, the optimal transmission rate should be a constant which is called the *wOPT rate*. Based on this discovery, the offline delay minimization problem has been solved. Then, an online scheduling algorithm based on the *wOPT rate* was proposed, and its performance was proved to be bounded by a ratio 1.16 to the offline optimal performance in terms of transmission delay for empty initial battery case. In case the battery starts with an arbitrary initial energy, the online algorithm performance was evaluated by simulations. The results verified the efficiency of the proposed algorithm.

APPENDIX

A. Proof of Theorem 2

The produced curve consists of line segments set in Line 5, 9, 14, 17 and 22. We now show that they are all optimal. It is obvious that if Algorithm DMSP-LARGEBATTERY returns in the **if** statement of Line 4, the produced departure curve is optimal according to Theorem 1.

We now show that the line segment set in Line 9 is optimal by showing (1) its slope is optimal and (2) its ending point is optimal. **(1)** The slope in Line 9 is obvious r_s. We now prove it is the optimal rate. Suppose the optimal second rate changing point is t_2^{opt}. According to Lemma 2, t_2^{opt} must be an arrival point, let it be a_k; according to Lemma 3, packets $P_1, P_2, \ldots, P_{k-1}$ must have been completely delivered before a_k. Therefore, the optimal solution uses the minimum energy to deliver packets $P_1, P_2, \ldots, P_{k-1}$ before a_k, because only in such a way the maximum energy can be used to transmit the rest packets after a_k to minimize the completion time. According to the discussion about the dual problem right after Theorem 1, the *wOPT rate* r_s is the optimal rate. **(2)** We now prove $t_2^{opt} = t_2$ by contradiction. Suppose $t_2^{opt} < t_2$, since t_2^{opt} is an arrival point, let $a_o = t_2^{opt}$. According to Line 7, we have $a_o - \frac{\sum_{j=0}^{o-1} B_j}{r_s} < a_k - \frac{\sum_{j=0}^{k-1} B_j}{r_s}$. Therefore, $\frac{\sum_{j=o}^{k-1} B_j}{a_k - a_o} < r_s$. This means the optimal rate decreases at t_2^{opt}, which contradicts Lemma 1. Suppose $t_2^{opt} > t_2$, then according to Line 7, point $(t_2, \sum_{j=0}^{k-1} B_j)$ will be on the right of line segment $(a_o - \frac{\sum_{j=0}^{o-1} B_j}{r_s}, 0) - (t_2^{opt}, \sum_{j=0}^{o-1} B_j)$, violating the causality constraint. Hence, $t_2^{opt} = t_2$.

We now show the line segments set in the **while** loop, *i.e.,* in Line 14 and 17, is optimal. We prove this by induction. In the first loop, all line segments before $a_k = t_2$ is optimal, which serves as the base. We assume, for any loop, all line segment before a_k is optimal, and we need to show the segment drawn in current loop of Line 14 or 17 is also optimal. It is easy to see that if $r_{nowpt} < r_{min}$, where r_{nowpt} is calculated by Eq. (11), it is optimal to minimize the completion time. Therefore line segment drawing in Line 14 is optimal. We next show the line segment drawing in Line 17 is also optimal. For the sake of contradiction, suppose $a_{k_{new}}$ is not the optimal ending, instead, $a_o \neq a_{k_{new}}$ is optimal. According to Line 17, we have $\frac{\sum_{j=k}^{k_{new}-1} B_j}{a_{k_{new}}-a_k} < \frac{\sum_{j=k}^{o-1} B_j}{a_o-a_k}$. If $o < k_{new}$, then $\frac{\sum_{j=o}^{k_{new}-1} B_j}{a_{k_{new}}-a_o} < \frac{\sum_{j=k}^{o-1} B_j}{a_o-a_k}$, which means the rate in a subsequence time duration $[a_o, a_{k_{new}}]$ is lower than that in duration $[a_k, a_o]$, contradicting Lemma 1. Otherwise, we have $o > k_{new}$, then point $(a_{k_{new}}, \sum_{j=0}^{k_{new}-1} B_i)$ is on the right of line $(a_k, \sum_{j=0}^{k-1} B_i) - (a_o, \sum_{j=0}^{o-1} B_i)$, violating the causality constraint.

Segment set in Line 22 is optimal because it is computed by Eq. (11). Therefore, all line segments set in this algorithm is optimal.

The dominated operation in this algorithm is the **while** loop. Inside the loop, the computation of r_{min} in Line 11 dominates, which takes n steps to calculate and find the minimum value. The **while** loop repeats one most n times, because each iteration k increases at least 1, and loop terminates after $k \geq n$. Therefore the time complexity of this algorithm is $O(n^2)$.

B. Proof of Theorem 3

We first prove two lemmas, then present the proof.

Lemma 6: In an optimal schedule, a sending phase must have its transmission rate $r^{opt} \geq r_s$ if there is a preceding charging phase.

Proof: We prove by contradiction. Suppose $[t_1, t_2]$ is the first sending phase with $r_{opt} < r_s$ and there is a charging phase $[t_0, t_1]$ ahead. It is clear that the amount of data transmitted by the given optimal schedule in $[t_1, t_2]$ is $B = r^{opt}(t_2 - t_1)$. Suppose the transmitter has remain energy E_2 in battery at t_2. Now, we consider the single cycle scheduling problem to transmit B data in $[t_0, t_2]$ and maximizes the remain energy at t_2. By the discussion of the dual problem at the end of Section IV, the maximum remain energy will be obtained by using rate r_s in the transmission. We now show it is feasible to modify the current schedule accordingly. Obviously, this modification will enlarge the charging phase, shrink the sending phase and increase the transmission rate in it. This is feasible, because, any portion of the B data is now delivered no earlier than its original delivery time, violating no causality constraint of packet arrival. After the modification, the remain energy at t_2 is increased, which obviously supports shorter transmission later. This is a contradiction since the optimal solution is modified to a even better result. □

Lemma 7: In an optimal rate schedule, a sending phase must have its transmission rate $r^{opt} \leq r_s$ if there is a subsequence charging phase.

Proof: We prove by contradiction also. Suppose $[t_1, t_2]$ is the first sending phase with $r_{opt} > r_s$ and there is a following charging phase $[t_2, t_3]$. Now, we consider the single cycle scheduling problem to transmit the same data in $[t_1, t_3]$ and maximizes the remain energy at t_3. Using r_s in transmission is the optimal solution, which enlarge the sending phase and shrink the charging phase. This modification is feasible, because firstly energy consumed in the sending is reduced according to the convexity of the power-rate function, and secondly any portion of data is now delivered no earlier than its original delivery time. This is a contradiction since the optimal solution is modified to a even better result. □

By the above two lemmas, in the optimal rate schedule, a sending phase must have its transmission $r^{opt} = r_s$ if it follows a charging phase and is followed by another charging phase. Thus, rate in every cycle is r_s before the very last charging phase.

C. Proof of Theorem 4

Let $B^{dms}(t)$ and $B^{opt}_{md}(t)$ denote the departure curve by Algorithm DMSP and the re-arranged optimal departure curve respectively. We now focus on the parallelograms that bound $B^{opt}_{md}(t)$ and $B^{dms}(t)$. Assume the former parallelogram starts at time t^{OPT}, while the later starts at t^{DMS}. We will first show that $t^{OPT} = t^{DMS}$ and then show the two transmission delays are the same.

Case 1: In case $t^{OPT} = 0$, there is no energy overflow in the optimal solution. Let parallelogram $C'D'E'F'$ bound $B^{opt}_{md}(t)$, where $E' = O$. Then, the arrival curve $A(t)$ must be on the left of $C'F'$, because $A(t)$ must be on the left of $B^{opt}_{md}(t)$ according to the causality constraint, and $B^{opt}_{md}(t)$ must be on the left of $C'F'$ since $C'D'E'F'$ bound $B^{opt}_{md}(t)$. Hence, we have $\frac{E_b}{p} \geq a_i - \frac{\sum_{j=0}^{i-1} B_j}{r_a}$, for $\forall i \in [1, n]$. As a result we get $t_0 = \frac{E_b}{p}$ in Line 3, which means $t^{DMS} = 0$.

Case 2: When $t^{OPT} > 0$, obviously, we have $t^{OPT} \leq t^{DMS}$ because our solution can not surpass the optimal one. To prove $t^{OPT} = t^{DMS}$, we need only to show $t^{OPT} \geq t^{DMS}$ as well. Because $t^{OPT} > 0$, there is energy overflow in the optimal solution, hence, $A(t)$ is partially on the right of the $C'F'$. Therefore $\frac{E_b}{p} \leq \max_i\{a_i - \frac{\sum_{j=0}^{i-1} B_j}{r_a}\}$ and $t_0 = a_k - \frac{\sum_{j=0}^{k-1} B_j}{r_a}$ for some k. Obviously, $t^{DMS} = t_0 - \frac{E_b}{p}$ is the leftmost position of any feasible parallelogram, including the optimal one, therefore we have $t^{DMS} \leq t^{OPT}$.

As a conclusion, we must have $t^{OPT} = t^{DMS}$. We next show that the transmission delay of our proposed algorithm is optimal, *e.g.,* $B^{opt}_{md}(t)$ and $B^{dms}(t)$ are with the same delay.

Assume t_l is the beginning of the last cycle by the proposed algorithm (not the optimal algorithm), then we use the following three steps to prove the produced schedule is optimal. First, we show that the optimal schedule uses only r_s before t_l. It is obviously true if the optimal schedule uses r_s for the entire duration. If the optimal schedule uses a higher rate than r_s, let it begin at t_h, than t_h must be a packet arrival point according to Lemma 2. The data arrived after t_h must be less than $\frac{E_b r_s}{2^{r_s}-1}$, this is because at most E_b energy can be consumed in transmission and the transmission rate is large than r_s. According to Algorithm DMSP, the **while** loop exits before or at t_h, *e.g.,* $t_l \leq t_h$, so the optimal schedule uses only

r_s before t_l. Second, we prove our online schedule is optimal before t_l. Recall the lost periods $t^{OPT} = t^{DMS}$ for both optimal schedule and our online schedule and there is no energy overflow after such periods. Therefore, by the time t_l, the two schedules harvest the same amount of energy and transmits the same amount of data, resulting in the same amount of remain energy in battery. Hence, our online schedule is optimal before t_l. Third, we demonstrate our online schedule is optimal after t_l. When Algorithm DMSP-LargeBattery is invoked in Line 14, the unsent data must be less than $\frac{E_b r_s}{2^{r_s}-1}$ which is the exit condition of the **while** loop, the battery capacity E_b is large enough to support delivery of these data. Since Algorithm DMSP-LargeBattery has already been proved to optimal to compute the transmission schedule, our online schedule is optimal after t_l.

In this algorithm, the time complexity depends on the execution of **while** loop and the invocation of DMSP-LargeBattery. Since each invocation of DMSP-LargeBattery takes $O(n^2)$ steps. Each iteration of the **while** loop ends when a packet arrives or no energy/data left. There are at most n packet arrival points. We now count how many possible points with empty battery. Because, the battery is always charged to full in case no energy left in the last iteration as in Line 10, and each full battery can delivery $\frac{E_b r_s}{2^{r_s}-1}$ data. Hence, at most $\frac{B\times(2^{r_s}-1)}{E_b r_s}$ points with empty battery. Now, we know, the **while** loop repeats at most $\min\{\frac{B\times(2^{r_s}-1)}{E_b r_s}, n\}$ iterations. Combine the two parts, and the time complexity is $\min\{\frac{B\times(2^{r_s}-1)}{E_b r_s}, n^2\}$.

D. Proof of Lemma 5

Transmitting B data at rate r_s takes $\tau_2^s = B/r_s$ time. Since the harvested energy is all consumed, $r_s = \log(1 + \frac{p\tau_1^s}{\tau_2^s})$, hence $\tau_1^s = \frac{B(2^{r_s}-1)}{r_s p}$ stands for the charging phase length that supports transmitting B data at rate r_s.

We draw the arrival curve $A(t)$, the modified departure curve $B^{online}(t)$ by DMSP-Online and the offline optimal departure curve $B^{offline}(t)$ by DMSP-LargeBattery on the *cumulative data-time* diagram, as in Fig. 5. Obviously, $B^{online}(t)$ consists of segment OL and segment LK which is with slope r_s. We claim that $A(t)$ *is on the left of* LK *and either 1) the two have one tangency point while* $\tau_1 > \tau_1^s$*, or 2)* $\tau_1 = \tau_1^s$. The argument is as follows. According to algorithm DMSP-Online, in the original online solution before modification, the last sending phase begins either 1) after a packet arrival (by Line 5), or 2) energy enough to delivery the remain data (by Line 7). Because after modification, the delay is not changed, so the last sending phase is not changed either. Hence the $A(t)$ and LK must 1) have one tangency point when $\tau_1 > \tau_1^s$ if Line 5 starts the last sending phase, or 2) $\tau_1 = \tau_1^s$ if Line 7 starts the last sending phase. Recall the steps of computing the offline optimal departure curve $B^{offline}(t)$ in Section V, *e.g.,* a line with slop r_s is moved to a position such that it has a tangency point to $A(t)$. Such position must be LK. Therefore, τ_1 must be the length for both charging phases, and $\tau_1 \geq \tau_1^s$.

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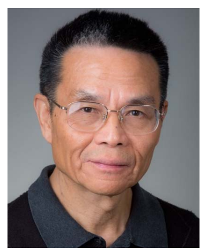

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