Optimal Wireless Power Transfer Scheduling for Delay Minimization

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Abstract*—Wireless power transfer (WPT) technique enables wireless charging/recharging, thus is a promising way to power wireless devices' transmissions. Because current WPT technique requires a wireless device to stop transmitting data when receiving power, and also because the received power in this way is limited, careful scheduling is needed to decide when the device should receive power and when it should transmit such that data can be efficiently transmitted. This paper assumes the most fundamental point-to-point White Gaussian Noise channel is used for data transmission and attempts to obtain an optimal scheduling such that a sequence of data packets can be transmitted with the minimum delay. It is discovered that, for all (energy receiving, data transmitting) cycles, except the last one, the optimal transmission rate should be a constant which is called the *wOPT rate*. Based on this discovery, this paper optimally solves the offline delay minimization problem. Then, an online heuristic scheduling algorithm is proposed, which either receives energy or transmits at the *wOPT rate*. Simulations have demonstrated its efficiency. The discovery of the *wOPT rate* reveals an essential property of WPT, thus is expected to make significant impact in the field of WPT.

I. Introduction

With the rapid development of mobile devices and the Internet of Things, current battery technique is becoming further and further from satisfactory. The current batteries are typically large in size, heavy in weight, low in capacity, and slow to charge. Wireless power transfer (WPT) provides an alternative option to overcome these disadvantages. Moreover, WPT technique enables devices to be battery-free, which has many potential applications in Internet of Things. WPT technique also provides a new approach toward the perpetual wireless sensor networks.

In industry, groups and companies have already been working on the commercialisation and standardization of WPT techniques. For instance, *Qi* [1] is a new wireless power transfer interface standard, which is becoming increasingly popular; *WiTricity* is a now the biggest company dedicated to wireless power transfer who holds an important patent for it [2]. A large number of multinational corporations such as Samsung, Huawei, Sony, Intel and Toyota have been involving in the *Qi* standard or *WiTricity* licensing. Their productions are being available quite recently on the market, such as wireless chargers for cell phones, or even cell phones with built-in wireless charging capability. According to an Intel executive, wireless charging will reach PCs in 2016 [3]. By then, laptops will be able to recharge when placed on tables, pads or surfaces supporting power delivery [3].

In academia, increasing interests are turning to WPT. In a point-to-point energy transfer experiment [4], wireless power of 3.5 mW has been harvested from the RF signals at distances of 0.6 meters. One of the most significant work is by V. Liu *et al.* [5]. They design and make a new type of battery-free device that communicates with each other by energy harvested from television broadcast signals. A most recent work by V. Talla *et al.* [6] builds the first *power over Wi-Fi* system that delivers power via commercially available Wi-Fi chipsets. Such system can provide far field wireless power without compromising the network's communication performance.

H. Ju *et al.* [7] study a new type of access point called H-AP, which provides wireless energy to user devices and collects information from them. Since the power transfer is in the downlink (DL) while the data transmission is in the uplink (UL), they propose a 'harvest-then-transmit' protocol to coordinate the two operations to maximize network throughput. Such problem is further studied in a large-scale wireless powered communication network by Y. L. Che *et al.* [8]. G. Yang *et al.* [9] further extend these works by assuming such H-APs are equipped with large number of antennas.

These related works either study the feasibility of WPT by designing hardwares or study the WPT problem from the point view of network throughput maximization. However, guaranteeing the maximum network throughput does not necessarily guarantee a specific user device's data transmission delay. In many real world applications, the data transmission delay is required as a part of quality of service (QoS) for time-sensitive applications.

In this paper, we investigate a fundamental scheduling problem for a wireless device such that a sequence of dynamically arrived data packets can be transmitted with the minimum delay. As previous work [7]–[9], we assume the 'harvest-then-transmit' protocol for the wireless transmitter device. The transmitter has to decide 1) when to receive energy and when to deliver data, 2) what transmission rate should be used to deliver data in each transmission period, 3) how often to repeat the (energy receiving, data transmitting) cycles? The ultimate goal is to minimize the completion time with all packets transmitted. We assume the most fundamental point-to-point single-user additive White Gaussian Noise (AWGN) channel for data transmission.

In our research, we need to address several challenges.

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According to Shannon-Hartley Theorem on wireless channel capacity, a low transmission rate is preferred to save energy, while a high rate is preferred to shorten transmission delay. Therefore, a major challenge lies on the trade-off between the following two strategies. On one hand, the more time to receive power, the more energy is charged to the device which allows a higher transmission rate and shorter time to deliver data. On the other hand, the more time to deliver data, the less energy is required and less time is needed to charge the device. To minimize the total time on receiving power and sending data, the best trade-off must be found.

Another major challenge is that the device battery has a limited capacity. When it is full, no more energy can be added; while it is empty, no data can be transmitted. Therefore, the transmitter must alternatively change its operations, from charging the battery to sending data, and vice versa. This adds one more difficult factor to our problem.

The contributions is summarized as follows.

- We formally define the delay minimization scheduling problem for the transmission channel with wireless power transfer capability.
- We discover that although the optimal time duration for each (energy receiving, data transmitting) cycle depends on the initial energy of the battery and sizes of packets, the optimal transmission rate for each cycle, except the last cycle, is constant and dependents on neither of them. Such rate is called the *wOPT rate*.
- Based on the *wOPT rate*, we design an optimal scheduling algorithm to solve the offline delay minimization problem. In the offline optimal solution, we determine for the transmitter the optimal switching point between receiving energy and sending data at *wOPT rate* until the very last cycle which needs higher rates to speed up the completion.
- For the online problem, we provide a heuristic strategy that switches between receiving energy and sending data at *wOPT rate* for the entire period.
- The discovery of the *wOPT rate* reveals an essential property of WPT, thus is expected to make significant impact on other scheduling problems in the field of WPT.

The organization of this paper is as follows. Section II formally defines the delay minimization scheduling problem. The notion of *wOPT rate* is introduced in Section III. Section IV studies the offline problem, and utilizes the *wOPT rate* to optimally solve this problem. An online heuristic algorithm is proposed in Section V, followed by simulations that show its efficiency. Related works are introduced in Section VI. Section VII concludes this paper.

II. Problem Formulation

A. System model

Suppose an AWGN channel consists of a wireless powered transmitter and a receiver. Let $P = \{P_1, P_2, \ldots, P_n\}$ be a set of n packets waiting in a queue to be transmitted from the transmitter to the receiver, as shown in Fig. 1. Each packet P_i has a size B_i, an arrival time a_i, and is denoted as $P_i(B_i, a_i)$. We assume each packet has a distinct arrival time such that $a_1 < a_2 < \ldots < a_n$. If two or more packets arrive at the same time, we combine them into a single packet with its size being the sum of all sizes in these packets. The transmission of packet P_i can start only after its arrival time a_i. This is called the *causality constraint* [16].

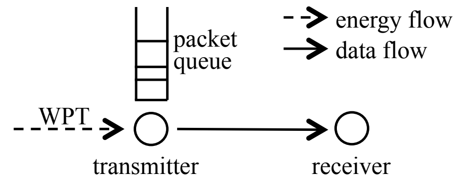

Fig. 1. A wireless powered transmission system

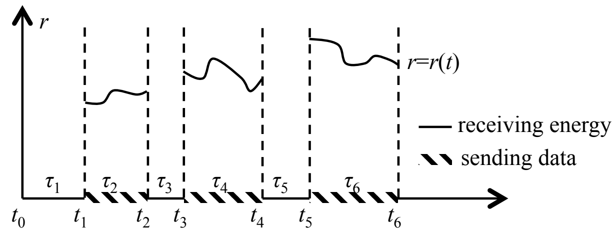

Fig. 2. Charging phases, sending phases and cycles

The wireless transmitter is capable of receiving energy wirelessly via WPT technique. When receiving energy, its battery is charged; the received energy then is used to send data at a later time. We therefore define the *charging phase* and *sending phase*, respectively. These two phases form a (energy receiving, data transmitting) cycle. Following previous work [7]–[9], a transmitter can either be in the *charging phase* or be in the *sending phase*, but not in both. The transmitter switches between the two phases alternatively according to a scheduling algorithm until all data packets are completely delivered.

Suppose there are m cycles. Thus, there are $2m$ phases and $2m$ switches, which occur at time instances $\{t_1, t_2, \ldots, t_{2m}\}$, $0 < t_1 < \ldots < t_{2m}$. The $2m$ phases are labeled from 1 to $2m$. Phase i starts from time t_{i-1} and ends at t_i. Its length is denoted as τ_i, e.g. $\tau_i = t_i - t_{i-1}$. Note that we assume $t_0 = 0$. When no ambiguity arises, we also use the notation τ_i to denote Phase i. Phase $2i-1$ (τ_{2i-1}) is a charging phase, and Phase $2i$ (τ_{2i}) is a sending phase, $i = 1, 2, \ldots, m$. Note, we assume the last phase is a *sending phase*, this is because if the last phase is otherwise a *charging phase*, we can delete it without affecting the transmission completion time. Therefore, the set $\{t_i\}$ is called the *switch points*, meaning phase changes at such time instances, as shown in Fig. 2.

Let p be the energy transfer speed (amount of energy received per second) during the charging phases. The speed is assumed to be a constant speed.

B. Problem formulation

Let $H(t)$ be the total energy charged into the battery in duration $[0, t]$. We can calculate $H(t)$ as follows.

$$H(t) = \begin{cases} \sum_{j=0}^{i-1} \tau_{2j+1} p & \text{for} \quad t_{2i-1} \le t < t_{2i} \\ \sum_{j=0}^{i-1} \tau_{2j+1} p + (t - t_{2i}) p & \text{for} \quad t_{2i} \le t < t_{2i+1} \end{cases}$$

During the *sending phases*, it is assumed that the transmitter can adaptively change its transmission rate.

Definition 1. *The transmission rate function $r(t) : \mathbb{R}_{\ge 0} \to \mathbb{R}_{\ge 0}$ is defined as the transmission rate at time t.*

We hence denote the transmission rate as a function of time

$$r(t) \begin{cases} = 0 & t \text{ in } \tau_1, \tau_3, \dots, \tau_{2m-1} \\ \ne 0 & t \text{ in } \tau_2, \tau_4, \dots, \tau_{2m} \end{cases} \tag{1}$$

The transmission rate $r(t)$ is related to transmission power $p_t(t)$ through a function Eq. (2) in a single user point-to-point transmission channel [10]–[13], [15]–[18].

$$r(t) = \log(1 + p_t(t)) \tag{2}$$

As a result, the total amount of data transmitted during $[0, t]$ can be calculated by the following integration,

$$B(t) = \int_0^t r(x)\,\mathrm{d}x \tag{3}$$

$$= \begin{cases} \sum_{j=1}^{i} \int_{t_{2j-1}}^{t_{2j}} r(x)\,\mathrm{d}x & t_{2i} \le t < t_{2i+1} \\ \sum_{j=1}^{i-1} \int_{t_{2j-1}}^{t_{2j}} r(x)\,\mathrm{d}x + \int_{t_{2i-1}}^{t} r(x)\,\mathrm{d}x & t_{2i-1} \le t < t_{2i} \end{cases} \tag{4}$$

Thus, the causality constraint can be expressed as

$$B(t) \le \sum_{i: a_i < t} B_i, \quad \forall t > 0. \tag{5}$$

According to Eq. (2) and (1), we have the transmission power as $p_t(t) = 2^{r(t)} - 1$. The total energy consumed during $[0, t]$ can be calculated by the following integration,

$$E(t) = \int_0^t p_t(x)\,\mathrm{d}x \tag{6}$$

$$= \begin{cases} \sum_{j=1}^{i} \int_{t_{2j-1}}^{t_{2j}} p_t(x)\,\mathrm{d}x & t_{2i} \le t < t_{2i+1} \\ \sum_{j=1}^{i-1} \int_{t_{2j-1}}^{t_{2j}} p_t(x)\,\mathrm{d}x + \int_{t_{2i-1}}^{t} p_t(x)\,\mathrm{d}x & t_{2i-1} \le t < t_{2i} \end{cases} \tag{7}$$

Suppose the battery capacity is E_b and the initial energy in battery is E_0. In any time instance t, the total energy consumed $E(t)$ can not exceed the received energy $H(t)$ plus the initial energy E_0 in the battery, this is called the *energy constraint*.

$$E_0 + H(t) - E(t) \ge 0, \quad \forall t \in [0, t_{2m}], \tag{8}$$

where $E_0 + H(t) - E(t)$ is also called the *remain energy* in the battery. Such a remain energy can not exceed the battery capacity.

$$E_0 + H(t) - E(t) \le E_b, \quad \forall t \in [0, t_{2m}], \tag{9}$$

Let $T = t_{2m}$ be the end of the last phase, then at T, all packets must have been completely transmitted. This is called *load constraint* expressed by the following equation,

$$B(T) = \sum_{i=1}^{n} B_i. \tag{10}$$

Time T is called the *transmission delay* or *completion time*.

In this paper, we want to determine total number of phases $2m$, all the switch points $t_1, t_2, \dots t_{2m}$ and the transmission rate $r(t)$ in all *sending phases*, such that the required constraints (5)(8)(9)(10) are satisfied and the *transmission delay* is minimized. The formal definition is given below.

Definition 2 (Delay Minimization Scheduling problem, DMS problem)**.** *Given a set of packets P and a wireless power transmission system described above, the delay minimization transmission scheduling problem is to determine the number of cycles m, all the switch points $t_1, t_2, \dots t_{2m}$ and the transmission rate $r(t), 0 \le t \le T$ such that the causality constraint Eq. (5), the energy constraint Eq. (8), the battery capacity constraint Eq. (9) and the load constraint Eq. (10) are satisfied and the transmission delay T is minimized.*

The transmission rate $r(t)$ in the optimal solution for this problem is denoted as $r^{opt}(t)$ and referred to as the optimal rate schedule.

III. The *wOPT rate*

This section concentrates on a simplified scenario where only one packet is in P and battery has sufficiently large capacity, namely DMS-1 problem. A surprising result is that, in the optimal solution that minimizes the completion time, the transmission rate depends on neither the packet size nor the initial energy when the initial energy is below a criterion (Theorem 1).

Suppose in the DMS-1 problem, the only packet arrives at time 0 and has a size B. Imagine that if no WPT is available, in order to minimize transmission delay, we can use a single rate throughout the transmission to delivery data. The correctness of such a single-rate transmission lies in the convex property of the power-rate function Eq. (2). More specifically, if two transmission rates are used, we can always find a single rate in between that delivers the same amount of data in a shorter time. Detailed proof can be found in previous works [10], [11], [15], [16]. Suppose such single rate is r, then the completion time $\tau = \frac{B}{r}$. All energy should be used up at the end, thus the transmission power is $p_t = \frac{E_0}{\tau} = \frac{E_0}{B} r$, where E_0 is the initial energy. Combining with Eq. (2), we have

$$\log(1 + \frac{E_0}{B} r) = r.$$

This equation can be solved to obtain the value r,

$$r = -\frac{\mathcal{W}\left(-\frac{B \ln 2}{E_0 2^{\frac{B}{E_0}}}\right)}{\ln 2} - \frac{B}{E_0}, \tag{11}$$

where function $\mathcal{W}(z)$ is called the Lambert W function [20], which has the following property,

$$\mathcal{W}(z)e^{\mathcal{W}(z)} = z.$$

Therefore the completion time can be computed as follows,

$$\tau = B/r = -B/\left(\frac{\mathcal{W}\left(-\frac{B\ln 2}{E_0 2^{\frac{B}{E_0}}}\right)}{\ln 2} + \frac{B}{E_0}\right). \tag{12}$$

Note, we can see from Eq. (11) and (12) that both the transmission rate r and the completion time τ depend on B and E_0.

Now imagine the wireless device has another option that it can receive wireless power supply to charge the battery. Since the battery is sufficiently large, it is easy to see that one *charging phase* and one *sending phase* is enough, e.g. $m = 1$. This is because if $m > 1$, we can always combine all *charging phases* and all *sending phases* together without affecting the completion time. We assume the only *charging phase* is with length τ_1, and the only *sending phase* is with length τ_2. In order to minimize completion time, we must use a single transmission rate in the *sending phase*. Let it be r.

As a result, the total amount of energy in the battery by the end of the *charging phase* is $E_0 + p\tau_1$; the total amount of energy consumed in the *sending phase* is $\tau_2(2^r - 1)$. They must be equal, thus we have

$$E_0 + p\tau_1 = \tau_2(2^r - 1). \tag{13}$$

Since all data B is completely delivered, we have

$$\tau_2 = \frac{B}{r}. \tag{14}$$

As a result, Eq. (13) can be re-written as

$$E_0 + p\tau_1 = \frac{B}{r}(2^r - 1). \tag{15}$$

Multiplying p on both sides of Eq. (14), we get

$$p\tau_2 = p\frac{B}{r}. \tag{16}$$

Adding Eq. (15) to Eq. (16), we have

$$E_0 + p\tau_1 + p\tau_2 = \frac{B}{r}(2^r - 1 + p). \tag{17}$$

We further have

$$\tau_1 + \tau_2 = \frac{B(2^r - 1 + p)}{rp} - \frac{E_0}{p}. \tag{18}$$

Eq. (18) shows that the total completion time $T = \tau_1 + \tau_2$ is a function of variable r, which also depends on the data size B and the initial energy E_0.

We define the function $T(r)$ as

$$T(r) = \tau_1 + \tau_2 = \frac{B(2^r - 1 + p)}{rp} - \frac{E_0}{p} \tag{19}$$

Now, an interesting problem is to find the value of r such that the delay time $T(r)$ is minimized for a given initial energy E_0 and data size B. To find this value, let $T(r)' = (\frac{B(2^r-1+p)}{rp} - \frac{E_0}{p})' = 0$. We have

$$\left(\frac{2^r - 1 + p}{r}\right)' = 0.$$

Solving this equation, we get,

$$r = \frac{\mathcal{W}(\frac{p-1}{e}) + 1}{\ln 2}. \tag{20}$$

Letting $w = \mathcal{W}(\frac{p-1}{e})$, we define r_s as follows

$$r_s = \frac{w + 1}{\ln 2}. \tag{21}$$

We called r_s in Eq (21) the *wOPT rate*, standing for *the optimal transmission rate by wirelessly powered transmitter*. When data is transmitted at the *wOPT rate* r_s, the delay time $T(r)$ is minimized, and the minimum value is $T(r_s)$. The *wOPT rate* r_s is a constant because it depends only on $w = \mathcal{W}(\frac{p-1}{e})$, which depends only on p and p is a constant.

Note that r_s depends on neither data size B nor battery initial energy E_0. However, the two phase lengths τ_1 and τ_2 depend on both. They can be calculated as follows.

$$\tau_2 = \frac{B}{r_s}, \quad \tau_1 = \frac{\frac{B}{r_s}(2^{r_s} - 1) - E_0}{p}. \tag{22}$$

From Eq. (22) we can see that, the wireless power transfer is necessary (τ_1 is positive) only when the initial energy E_0 is small $E_0 \le \frac{B}{r_s}(2^{r_s} - 1)$. When $E_0 > \frac{B}{r_s}(2^{r_s} - 1)$, energy in battery is already sufficient to send all B data, the wireless power transfer is unnecessary. Thus, the minimum delay is computed by Eq. (12).

We hence summarize and emphasize our conclusion of this section in Theorem 1, whose correctness follows directly from the above discussion.

Theorem 1. *If the initial energy* $E_0 \le \frac{B}{r_s}(2^{r_s} - 1)$*, then the optiaml solution of the DMS-1 problem consists of a charging phase and a sending phase. The transmission rate in the sending phase is a constant* r_s *by (21) although the length of the two phases depend on* E_0 *and* B*, by (22). If* $E_0 > \frac{B}{r_s}(2^{r_s} - 1)$*, no charging phase is needed, thus the completion time can be computed by (12).*

In fact, the notion *wOPT rate* is so important that not only it is the unique optimal rate to achieve the minimum delay, but it is also the optimal rate for the dual problem. The dual problem asks to maximize the remain energy when transmitting the packet before a given deadline. It is not difficult for readers to follow similar approaches discussed above to solve the dual problem. We omit details here, but will use this conclusion directly in later sections.

In later sections, we will show that although the *wOPT rate* is derived from the simple scenario, it indeed plays an important role in the general scenario. Therefore, we conclude that the discovery of the *wOPT rate* reveals an essential property of WPT.

IV. Offline Optimal Solution for the Delay Minimizaiton Scheduling Problem

In this section, we study the offline delay minimization scheduling problem, where information about all packets in set P is known, including their arrival time and their sizes. We first investigate the problem where battery capacity E_b is sufficiently large and design an optimal algorithm by utilizing the *wOPT rate*. We then solve the general problem by taking the battery capacity E_b into consideration, where E_b can be an arbitrary small positive value.

A. An optimal solution for the large battery DMS problem

In this subsection, we allow n packets in set P, $P = \{P_1, P_2, \ldots, P_n\}$ and $P_i(B_i, a_i)$, but still require the battery capacity to be sufficiently large.

A large battery capacity allows us to combine all the *charging phases* together into one *charging phase* and does not affect the completion time. We thus focus on finding the optimal one (energy receiving, data transmitting) cycle solution.

If all packets arrive immediately after time 0, we can treat them as a single packet with size $B = \sum_{1\le i\le n} B_i$ and the *wOPT rate* r_s is still the optimal rate in the *sending phase* as discussed in the last subsection. However, if some packets arrive very late, then at some time point t, the transmitter is forced to stop because all arrived data have been transmitted and some packets in P have not arrived yet. We could charge more energy while we are waiting for these packets to arrive. This extra energy allows us to use a higher rate than r_s to shorten the completion time.

Before we present the optimal algorithm that produces the minimum completion time T, we would like to state some properties of the optimal solution, i.e. the optimal rate schedule, should have. Since we focus on the one cycle solution, the following lemmas are about the transmission rate in the only *sending phase*.

Lemma 1. *The optimal rate schedule $r^{opt}(t)$ is a non-decreasing function until the completion time T.*

Lemma 2. *The optimal rate schedule $r^{opt}(t)$ increases only at a packet arrival time $a_i, 1 \le i \le n$.*

Lemma 3. *The optimal rate schedule $r^{opt}(t)$ increases only when all arrived data has been transmitted.*

Lemmas similar to Lemmas 1, 2 and 3 have been known in the literature for energy efficient wireless transmissions [10], [11] and energy harvesting wireless transmissions [15], [16], [19]. We omit the proofs because they can be proved by similar arguments. Interested readers are suggested to refer to [19] for more details.

We now introduce the *cumulative data-time* diagram [13]. Let $A(t) = \sum_{i:a_i\le t} B_i$ denote the total amount of bits that have arrived in time interval $[0, t]$. The curve of function $A(t)$ on the *cumulative data-time* diagram is called the *arrival curve*. Obviously, the *arrival curve* is with an up-stair-like shape, as depicted in Fig. 3. Similarly, we define the departure curve $B(t)$, which is the actual amount of data leaving the system (transmitted) during $[0, t]$. It is easy to see that a feasible departure curve $B(t)$ must be on the right side of the arrival curve $A(t)$ because of the causality constraint. Furthermore, the slope of $B(t)$ is actually the rate schedule $r(t)$. Let $B^{opt}(t)$ be the optimal departure curve, then, the determination of $B^{opt}(t)$ immediately leads to the determination of $r^{opt}(t)$. We therefore focus on $B^{opt}(t)$.

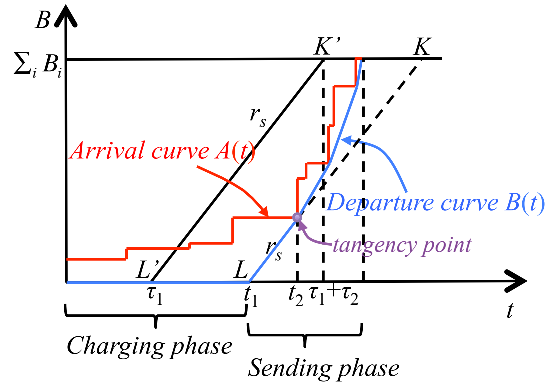

Fig. 3. In the *cumulative data-time* diagram, a feasible departure curve must be on the right side of the arrival curve. The slopes of the departure curve is the transmission rates. The optimal departure curve for the corresponding DMS-1 problem can be represented by the line segment $L'K'$. If $L'K'$ is on the right side of the arrival curve, we are done. Otherwise, we move $L'K'$ right to a new position LK such that the arrival curve is on its left side and there is a tangency point. From this tangency, we iteratively find the optimal line segment.

The high level idea to solve the large battery DMS problem is as follows. In the *cumulative data-time* diagram, i.e. Fig. 3, we draw a line segment with slope r_s connecting point $(\tau_1, 0)$ and point $(\tau_1 + \tau_2, B)$, where $B = \sum_{1\le i\le n} B_i$, while τ_1 and τ_2 are calculated by Eq. (22). As in Fig. 3, such line segment is $L'K'$, which represents the optimal rate schedule to transmit an amount of B data as discussed in the last section. If line segment $L'K'$ is on the right of the arrival curve $A(t)$, then such rate schedule is feasible and minimizes the completion time, thus we are done. Otherwise, we move this line segment towards its right, stop moving as soon as it is on the right of the arrival curve, i.e., LK in Fig. 3. Obviously, there is a tangency point on LK. The rightward movement suggests that the transmission starts at a later time $t_1, t_1 > \tau_1$, such that we get more energy charged into the battery and therefore can use a higher-slope line segment to minimize the completion time. We take this tangency point as the start point, take the remain energy in battery as E_0 and take the total amount of unsent packets as B, then we compute r_{nowpt} by Eq. (11) to minimize the completion time. If the line starting from this tangency point with slope r_{nowpt} is on the right of the arrival curve, we are done. Otherwise, we connect this tangency point and every corner of the arrival curve to find the lowest-slope line segment. We now take the ending point of this line segment as a start point and repeat this process until all packets are finished.

We present formal steps of this method in Algorithm DMSP-LargeBattery. Line 3-6 test whether line segment $L'K'$ is on the right of $A(t)$. Line 7-9 directly compute the position of line segment LK. The **while** loop repeatedly computes r_{nowpt} and the lowest-slope line segment.

Algorithm 1 DMSP-LargeBattery

```
1:  Set $B_0 = 0$ for loop purpose
2:  Let $\tau_1$ and $\tau_2$ be calculated by Eq. (22).
3:  $t_1 = \max_i(a_i - \frac{\sum_{j=0}^{i-1} B_j}{r_s})$
4:  if $t_1 < \tau_1$ then
5:      return line segment $(\tau_1, 0) - (\tau_1 + \tau_2, B)$
6:  end if
7:  $k = \arg\max_i(a_i - \frac{\sum_{j=0}^{i-1} B_j}{r_s})$
8:  $t_2 = a_k$
9:  Set line segment $(t_1, 0) - (t_2, \sum_{j=0}^{k-1} B_j))$
10: while $k < n$ do
11:     $r_{min} = \min_{k<i\le n} \frac{\sum_{j=k}^{i-1} B_j}{a_i - a_k}$
12:     Take the remain energy in battery as $E_0$ and take
        $\sum_{k\le i\le n} B_i$ as $B$, compute $r_{nowpt}$ by Eq. (11).
13:     if $r_{nowpt} < r_{min}$ then
14:         Set segment $(a_k, \sum_{j=0}^{k-1} B_i) - (a_k + \frac{\sum_{k\le i\le n} B_i}{r_{nowpt}}, B)$
15:         return all line segments
16:     else
17:         $k_{new} = \arg\min_{k<i\le n} \frac{\sum_{j=k}^{i-1} B_j}{a_i - a_k}$
18:         Set segment $(a_k, \sum_{j=0}^{k-1} B_i) - (a_{k_{new}}, \sum_{j=0}^{k_{new}-1} B_i)$
19:         $k = k_{new}$
20:     end if
21: end while
22: Take the remain energy as $E_0$ and take $B_n$ as $B$, compute
    $r_{nowpt}$ by Eq. (11).
23: Set line segment $(a_n, B - B_n) - (a_n + \frac{B_n}{r_{nowpt}}, B)$
24: return all line segments
```

Theorem 2. *Algorithm* DMSP-LargeBattery *produces the optimal departure curve $B^{opt}(t)$ for the offline DMS problem with a sufficiently large battery.*

Proof. The produced curve is consist of line segments set in Line 5, 9, 14, 18 and 23. We now show that they are all optimal. It is obvious that if Algorithm DMSP-LargeBattery returns in the **if** statement of Line 4, the produced departure curve is optimal according to Theorem 1.

We now show that the line segment set in Line 9 is optimal by showing (1) its slope is optimal and (2) its ending point is optimal. **(1)** The slope in Line 9 is obvious r_s. We now prove it is the optimal rate. Suppose the optimal second rate changing point is t_2^{opt}. According to Lemma 2, t_2^{opt} must an arrival point, let it be a_k; according to Lemma 3, packets $P_1, P_2, \ldots, P_{k-1}$ must have been completely delivered before a_k. Therefore, the optimal solution uses the minimum energy to deliver packets $P_1, P_2, \ldots, P_{k-1}$ before a_k, because only in such a way the maximum energy can be used to transmit the rest packets after a_k to minimize the completion time. According to the discuss about the dual problem right after Theorem 1, the *wOPT rate* r_s is the optimal rate. **(2)** We now prove $t_2^{opt} = t_2$ by contradiction. Suppose $t_2^{opt} < t_2$, since t_2^{opt} is an arrival point, let $a_o = t_2^{opt}$. According to Line 7, we have $a_o - \frac{\sum_{j=0}^{o-1} B_j}{r_s} < a_k - \frac{\sum_{j=0}^{k-1} B_j}{r_s}$. Therefore, $\frac{\sum_{j=o}^{k-1} B_j}{a_k - a_o} < r_s$. This means the optimal rate decreases at t_2^{opt}, which contradicts Lemma 1. Suppose $t_2^{opt} > t_2$, then according to Line 7, point $(t_2, \sum_{j=0}^{k-1} B_j)$ will be on the right of line segment $(a_o - \frac{\sum_{j=0}^{o-1} B_j}{r_s}, 0) - (t_2^{opt}, \sum_{j=0}^{o-1} B_j)$, violating the causality constraint. Hence, $t_2^{opt} = t_2$.

We now show the line segments set in the **while** loop, i.e., in Line 14 and 18, is optimal. We prove this by induction. In the first loop, all line segments before $a_k = t_2$ is optimal, which serves as the base. We assume, for any loop, all line segment before a_k is optimal, and we need to show the segment drawn in current loop of Line 14 or 18 is also optimal. It is easy to see that if $r_{nowpt} < r_{min}$, where r_{nowpt} is calculated by Eq. (11), it is optimal to minimize the completion time. Therefore line segment drawing in Line 14 is optimal. We next show the line segment drawing in Line 18 is also optimal. For the sake of contradiction, suppose $a_{k_{new}}$ is not the optimal ending, instead, $a_o \neq a_{k_{new}}$ is optimal. According to Line 17, we have $\frac{\sum_{j=k}^{k_{new}-1} B_j}{a_{k_{new}} - a_k} < \frac{\sum_{j=k}^{o-1} B_j}{a_o - a_k}$. If $o < k_{new}$, then $\frac{\sum_{j=o}^{k_{new}-1} B_j}{a_{k_{new}} - a_o} < \frac{\sum_{j=k}^{o-1} B_j}{a_o - a_k}$, which means the rate in a subsequence time duration $[a_o, a_{k_{new}}]$ is lower than that in duration $[a_k, a_o]$, contradicting Lemma 1. Otherwise, we have $o > k_{new}$, then point $(a_{k_{new}}, \sum_{j=0}^{k_{new}-1} B_i)$ is on the right of line $(a_k, \sum_{j=0}^{k-1} B_i) - (a_o, \sum_{j=0}^{o-1} B_i)$, violating the causality constraint.

Segment set in Line 23 is optimal because it is computed by Eq. (11).

Therefore, all line segments set in this algorithm is optimal. □

B. An optimal solution for the general problem

This subsection studies the original DMS problem of Definition 2, in which the battery has a capacity of E_b. Unlike the solution in the previous subsection, multiple (energy receiving, data transmitting) cycles are needed in this general case.

We will show how to divide the time into cycles and how to determine the transmission rate for each cycle, but first we introduce some properties about the optimal rate schedule.

Lemma 4. *In an optimal rate schedule $r^{opt}(t)$, the rate in every cycle is r_s except the one in the last cycle.*

In the last cycle, Lemmas 1, 2 and 3 still hold.

Proof. We prove by contradiction. Suppose $[t_1, t_2]$ is the sending phase of the first cycle in which the optimal rate schedule $r_{opt} \neq r_s$ and is not the last cycle. Then, there must be another sending phase following. And in between the two phases, it is an energy receiving interval $[t_2, t_3]$.

It is clear that the amount of data transmitted by the given optimal schedule in $[t_1, t_2]$ is $B = r^{opt}(t_2 - t_1)$. Suppose the transmitter has remain energy E_3 in battery at t_3. Now, if we consider the single cycle scheduling problem to transmit B data in $[t_1, t_3]$ and maximizes the remain energy at t_3, then by the discussion of the dual problem in the last subsection, the maximum remain energy will be obtained by using rate r_s in the transmission. Obviously, this will lead to a better result than the given optimal schedule, which is a contradiction. $\square$

Although we have known the transmission rate in each cycle except the last one is r_s, we still need to determine the beginning and ending of cycles and the rate schedule in the last cycle.

In the following discuss, we assume the battery is initial empty, i.e., $E_0 = 0$. Any non-zero initial energy $E_0 \neq 0$ can be equivalently considered as if an empty battery being charged for a length of $\frac{E_0}{p}$ time. We therefore move the starting time earlier, and during the first $\frac{E_0}{p}$ time, no packet arrives such that charging battery is the only option, and by original starting time, there is E_0 energy in the battery.

According to Lemma 4, in any (energy receiving, data transmitting) cycle other than the last one, although the transmission rate is r_s in the *sending phase*, the effective transmission rate is less than r_s because it needs a *charging phase* to charge. It equals the amount of data B transmitted over $\tau_1 + \tau_2 = \frac{B(2^{r_s}-1+p)}{r_s p}$. The *effective transmission rate* is thus $r_a = \frac{B}{\tau_1+\tau_2} = \frac{r_s p}{2^{r_s}-1+p}$.

Lemma 5. *On the cumulative data-time diagram, any optimal departure curve $B^{opt}(t)$ can be re-arranged to be bounded by a parallelogram with bottom length $\frac{E_b}{p}$, side line slope r_a and height B.*

One such parallelogram is illustrated in Fig. 4 as $CDEF$. The bottom edge EF is with length $\frac{E_b}{p}$, and the side edges DE and CF are both with slope r_a, and the height is B.

Proof. In an optimal solution, it is possible some energy is wasted because of the battery capacity constraint. In case the battery is already full, any further energy harvested will be lost. The charging period when the battery is full is called *the lost period*, which does not contribute to the energy in battery. In an optimal solution, the lost period may be in any *charging phase*. We combine all the lost periods into a single one and put it at the beginning of time. Obviously, the completion time is not affected and the modified solution is still optimal.

Suppose $t \geq 0$ is the ending of the combined single lost period in the modified solution, then after t, no energy is wasted. Note that, at time t, the battery is empty since lost periods do not contribute to energy in battery.

For the example in Fig. 4, suppose $t = x(E)$ is the end of the lost period. Once a departure curve reaches a point on DE, the battery becomes empty and the transmitter must stop sending and the departure curve horizontally goes to the right. Whenever the departure curve reaches a point on CF, the battery is full, then it must switch to sending data since no energy wasted after E, therefore the curve goes up right. As a conclusion, any departure curve must be inside $CDEF$. $\square$

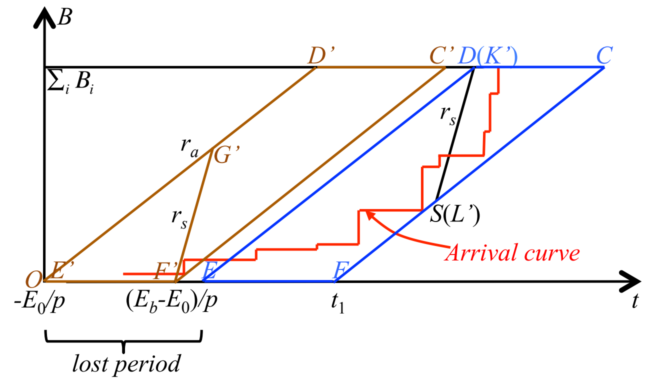

Fig. 4. From point O, system starts with an empty battery. There is a parallelogram $C'D'E'F'$ with point E' at O. $E'F'$ is with length $\frac{E_b}{p}$, which is the time length used to charge the battery from empty to full. Both $D'E'$ and $C'F'$ are with slope r_a, which is the maximum effective transmission rate. We move $C'D'E'F'$ to a new position $CDEF$ such that the arrival curve is the left side of CF. We then draw line from point D with the slope r_s, and make it DS. Now DS can be treated as $L'K'$ in Fig. 3 and apply the same technique to compute the optimal rate schedule.

Although we have known the optimal departure curve must be inside the parallelogram $CDEF$, the exact time t is unknown yet. We hence first determine the optimal position of the parallelogram $CDEF$, then determine the optimal departure curve inside this parallelogram.

The high level idea is as follows. If the parallelogram is at its leftmost position, i.e., $C'D'E'F'$, $C'F'$ is already on the right side of the arrival curve, then, $t = 0$ is the optimal position of the $CDEF$. Otherwise, we move the parallelogram to its right, stop moving as soon as CF is on the right of the arrival curve. We then draw line from point D with the slope r_s, which intersects with CF at point S. Before S, it is r_s in every *sending phase* according to Lemma 4. We can easily arrange a departure curve on the right side of the arrival curve that uses only rate r_s and make it go through point S. If DS is also on the right side of the arrival curve, then we set it as the last line segment of the departure curve. Otherwise, starting from S, we compute the departure curve by the technique developed in the last subsection.

We now give a formal pseudo code in Algorithm DMSP.

Theorem 3. *Algorithm* DMSP *produces the optimal departure curve for the general offline DMS problem.*

Proof. The produced curve is consist of line segments set in Line 4, 7, 11, 13 and 14. We show that they are all optimal.

We first prove the line segment set in Line 4 is optimal. It equals to prove that the optimal position of parallelogram is point F at $(t_1, 0)$, so point E is at $(t_1 - \frac{E_b}{p}, 0)$. By contradiction, assume the optimal position of point F is at $(t_1^{opt}, 0)$ instead. If $t_1^{opt} < t_1$, then point $(a_k, \sum_{j=0}^{k-1} B_j)$ is on the right of line segment FC, where k is computed in Line 3. Therefore, the optimal departure curve must go to the right of FC to reach such point. When the departure curve reaches a point on FC, the battery is already full, thus to reach a point

Algorithm 2 DMSP

1: Let $r_a = \frac{r_s p}{2^{r_s}-1+p}$
2: $t_1 = \max\{\frac{B}{r_a}, \max_i\{a_i - \frac{\sum_{j=0}^{i-1} B_j}{r_a}\}\}$
3: $k = \arg\max_i\{a_i - \frac{\sum_{j=0}^{i-1} B_j}{r_a}\}$
4: Set line segment $O-(t_1, 0)$ // battery is full at time t_1
5: Let $B_t = 0, B_{lim} = B - \frac{E_b r_s}{2^{r_s}-1}$
6: **while** $B_t \leq B_{lim}$ **do**
7:     Set line segments to transmit at r_s whenever both energy and data are available
8:     Charge until the battery is full
9: **end while** // these line segments go through point S
10: **if** DS is on the right of $A(t)$ **then**
11:     Set DS as the last line segment of the departure curve
12: **else**
13:     Taking S as start point, E_b as initial energy and assuming the battery is sufficiently large, we invoke DMSP-LARGEBATTERY to compute line segments
14:     Break line segments that are on the right of CF and move these pieces to on its left
15: **end if**
16: **return** all line segments

beyond FC, some energy must be wasted, and there is lost period after t_1. We can move such period t othe time before E thus move the parallelogram right, without affecting the completion time. Therefore $(t_1^{opt}, 0)$ is not the optimal position of point F. If $t_1^{opt} > t_1$, more energy is wasted before t_1^{opt} than t_1. Therefore, moving the parallelogram left will decrease the wasted energy, thus will not enlarge the completion time.

At the beginning of the last *sending phase*, the maximum amount of energy is E_b, the battery capacity. The maximum data can be transmitted is $\frac{E_b r_s}{2^{r_s}-1}$, using the *wOPT* rate r_s. Therefore, before the last cycle, at most $B_{lim} = B - \frac{E_b r_s}{2^{r_s}-1}$ data is delivered at rate r_s. So the schedule determined in **while** of Line 6 is optimal.

We now show the line segments introduced in Line 11, 13 and 14 is optimal. It is easy to see that if DS is on the right of $A(t)$, it is optimal to minimize the completion time according to the optimality property of the *wOPT* rate, hence the line segment set in Line 11 is optimal. Since DMSP-LARGEBATTERY has been proved to produce the optimal line segments that minimize the completion time, line segments set in Line 13 and 14 are optimal. □

V. ONLINE HEURISTIC AND SIMULATION RESULTS

Offline algorithms proposed in the last section require all the information about packets. However, in real world scenario, it is not easy to obtain such information. Hence, in this section, we design an online heuristic algorithm and conduct simulations to evaluate its efficiency.

A. Online Heuristic

According to Lemma 4, the optimal rate schedule in all cycles, except the last one, is r_s. However, the optimal switch point is unknown. We design Algorithm ONLINE-HEURISTIC to heuristically determine the switch points and set the rate in the last cycle to be r_s as well.

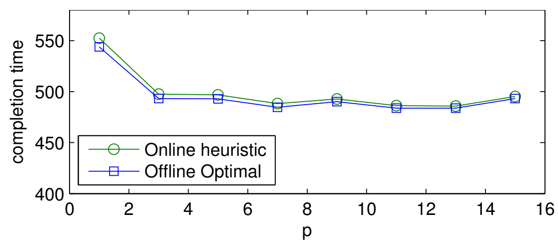

Fig. 5. The completion times when the energy transfer speed p changes from 1 to 15 with step 2.

Algorithm 3 ONLINE-HEURISTIC

Compute $r_s = \frac{w+1}{\ln 2}$
while more packets arrive **do**
    Transmit at r_s until no energy or no data left
    if no energy left **then**
        Charge until the battery is full
    else if no data left **then**
        Charge until the next packet arrives
    end if
end while

The idea in Algorithm ONLINE-HEURISTIC is simple: transmit at rate r_s as soon as energy and data are ready. We next show how this simple idea works.

B. Simulation Results

In this subsection, we implement the proposed Algorithm ONLINE-HEURISTIC and study its efficiency. Since there is no other algorithms focusing on the same DMS problem for a wireless powered device, we compare the online algorithm with the optimal offline Algorithm DMSP.

In simulations, 50 packets are generated following the Poisson arrival, where the arrival rate is set to be 1/10. Packet size is assumed to be a random variable following the uniform distribution $U(7, 10)$. The battery capacity E_b is between 800 and 1500, while the initial energy E_0 is assumed to be a value between empty $E_0 = 0$ and full $E_0 = E_b$ randomly. The energy transfer speed p is set between 1 and 15.

For every settings, we randomly generate 100 instances, and use the mean value of the results for comparison. The comparison results are illustrated in Fig. 5 and 6.

We can see from Fig. 5 that the larger the energy transfer speed p is, the shorter the completion time is. This is because when p is large, little time is taken to charge the battery, therefore the total time decreases. While p is small, it needs more careful effect to schedule the transmission. Thus the simple online heuristic fails to produce a solution close to the optimal. We can see the gap between the two curves is bigger when p is small.

From Fig. 6, we can conclude that the battery capacity does not have much impact on the algorithm performance. This is

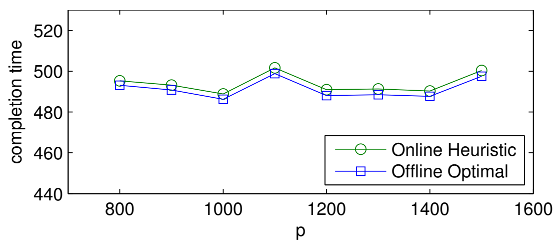

Fig. 6. The completion times when the battery capacity E_b changes from 800 to 1500 with step 100.

because both online heuristic and online optimal algorithms work in cycles. In a long run, the battery capacity plays unimportant role on the completion time.

From both figures, we can see that the online heuristic has a similar performance to the offline optimal solution. This is because the online heuristic is designed based on the optimality property Lemma 4 that states the optimal transmission rate is r_s, excepting in the last cycle.

VI. Related Work

E. Uysal-Biyikoglu *et al.* [10], [11] are among the first group to study the energy minimization problem for delivering a set of packets before a common deadline. They propose a *lazy* schedule to optimally solve the offline problem. Zafer and Modiano [12], [13] further generalize the problem to allow individual packet deadlines provided they follow the same order packets arrive. Most recently, Shan, Luo and Shen [14] solve the energy minimization problem that allows arbitrary individual packet deadlines. All these research works [10]–[14] do not consider energy harvesting.

J. Yang *et al.* [15], [16] consider the delay minimization problem for harvesting enabled channels assuming all harvesting events are pre-determined and take no time to receive the energy. They have obtained the optimal offline scheduling algorithm. A. Yener *et al.* extend their work to let the battery have a limited capacity [17], [18]. F. Shan *et al.* study the energy consumption minimization problem for an energy harvesting transmitter, they allow packets to have individual deadlines.

VII. Conclusions

In this paper, we have studied the delay minimization scheduling problem for a WPT device. We have obtained an optimal schedule such that a sequence of data packets can be transmitted with the minimum delay assuming the point-to-point AWGN channel. It was discovered that, for all (energy receiving, data transmitting) cycles, except the last cycle, the optimal transmission rate should be a constant which is called the *wOPT rate*. Based on this discovery, the offline delay minimization problem has been solved. Then, an online heuristic scheduling algorithm was proposed.

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